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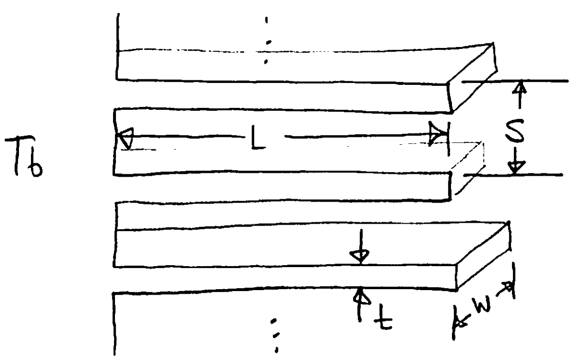
Overall Surface Efficiency (η_o)

The overall surface efficiency characterizes the performance of an array of fins.

$$\boxed{\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h A_t \theta_b}}$$

$q_t \equiv$ total heat transfer of fins & base of area (A_t)



Assuming:
1) $N \equiv$ number of fins in array
2) $A_f \equiv$ exposed area per fin
3) $A_b \equiv$ base area between fins
4) $\eta_f \equiv$ individual fin efficiency

$$A_t = N A_f + A_b$$

$$q_t = N \eta_f h A_f \theta_b + h A_b \theta_b$$

$$q_t = h\left[N \eta_f A_f + (A_t - N A_f)\right]\theta_b = h A_t \left[1 - \frac{N A_f}{A_t}(1-\eta_f)\right]\theta$$

$$\boxed{\eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f)}$$ $\Rightarrow$ Overall surface efficiency

We can now also calculate the thermal resistance of a fin arra

$$\boxed{R_o = \frac{\theta_b}{q_t} = \frac{1}{h A_t \eta_o}}$$ $\Rightarrow$ Thermal resistance of a fin array.

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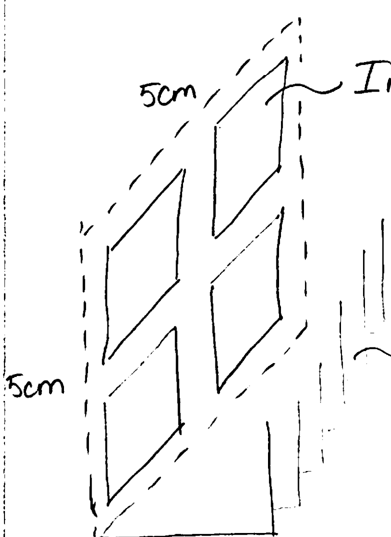
Example | CPU cooling

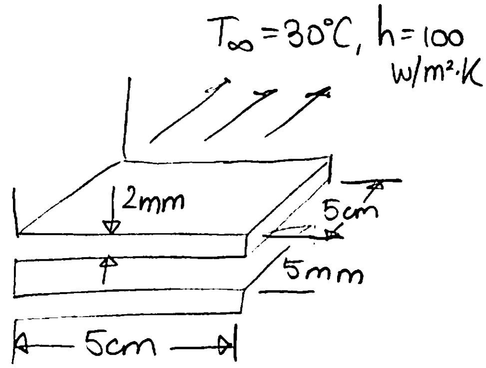


Intel Quad Core CPU's

$T_b = 70°C$

Finned Heat Sink (Copper)

$k_{cu} = 400 \ W/m \cdot K$



What is the increase in heat transfer due to the use of fins?

Due to the small fin thickness, we can ignore the fin edges, and use 1D fin analysis.

$$A_f = 2w\left(L + \frac{t}{2}\right) = 2(0.05m)\left(0.05m + \frac{0.002m}{2}\right) = 0.0051m^2$$

$$A_t = N(\underbrace{A_f}_{\text{Total fin Area}} + \underbrace{A_b}_{\text{Total Base Area}}) = \left[0.0051m^2 + (0.003m)(0.05m)\right]\underbrace{\left[\frac{0.05m}{0.005m}\right]}_{N}$$

$$A_t = 0.0525m^2$$

But wait, we didn't check the Biot #! $Bi_t = \frac{ht}{k} = \frac{(100)(0.002)}{400}$

$\boxed{Bi_t = 5\times10^{-4} < 0.1}$ $\Rightarrow$ OK!

We know for our individual fin:

$$\eta_{fin} = \frac{\tanh(mL)}{mL} = \frac{\tanh\left(\sqrt{\frac{hP}{kA}}L\right)}{\sqrt{\frac{hP}{kA}}\cdot L} = \frac{\tanh\left(\sqrt{\frac{2(0.05)(100)}{(400)(0.05)(0.002)}}(0.05)\right)}{\cdots} \left(= \frac{0.658}{0.79}\right)$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f) = 1 - \frac{10(0.0051)}{0.0525}(1-0.833) = 0.84$$

$q_t = hA_t\theta_b\eta_o = (100)(0.0525)(70-30)(0.84) = 176.4W$, $q_o = hA_o\theta_b = (100)(0.05)^2(40) = 10W$

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Steady Multi-Dimensional Heat Transfer (Shape Factor)

If a problem is steady (SS) and has isothermal surfaces, we can define whats called a "shape factor"

$$Q = Sk\Delta T$$

S → Obtained analytically or numerically

$$R_{th} = \frac{1}{kS}$$

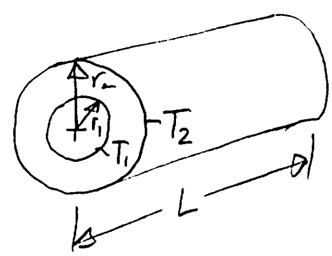
For example, let's see how it works out for a plane wall:

$$R_{wall} = \frac{L}{kA} = \frac{1}{kS} \Rightarrow \boxed{S_{wall} = \frac{A}{L}}$$

$$\boxed{Q = kA\frac{\Delta T}{L} = kA\frac{\partial T}{\partial x}}$$

⇒ Makes sense

How about a cylinder:



$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

$$\boxed{S_{cyl} = \frac{1}{kR_{cyl}} = \frac{2\pi L}{\ln(r_2/r_1)}}$$

⇒ Very simple and in general not needed for simple geometries as R_{th} is available & our previous analysis holds.

However for more complex shapes, it is a very usefull concept.

Table 5.4 Conduction shape factors: $Q = S k \Delta T$, $R_t = 1/(kS)$.

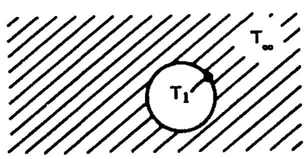
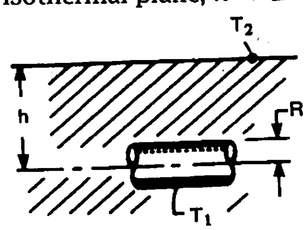
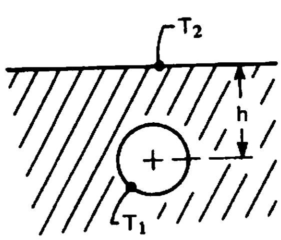
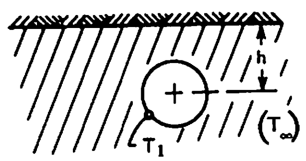
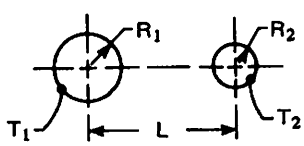
Situation	*Shape factor, S*	*Dimensions*	*Source*
1. Conduction through a slab	A/L	meter	Example 2.2
2. Conduction through wall of a long thick cylinder	$\dfrac{2\pi}{\ln(r_o/r_i)}$	none	Example 5.9
3. Conduction through a thick-walled hollow sphere	$\dfrac{4\pi(r_o r_i)}{r_o - r_i}$	meter	Example 5.10
4. The boundary of a spherical hole of radius R conducting into an infinite medium 	$4\pi R$	meter	Problems 5.19 and 2.15
5. Cylinder of radius R and length L, transferring heat to a parallel isothermal plane; $h \ll L$ 	$\dfrac{2\pi L}{\cosh^{-1}(h/R)}$	meter	[5.16]
6. Same as item 5, but with $L \longrightarrow \infty$ (two-dimensional conduction)	$\dfrac{2\pi}{\cosh^{-1}(h/R)}$	none	[5.16]
7. An isothermal sphere of radius R transfers heat to an isothermal plane; $R/h < 0.8$ (see item 4) 	$\dfrac{4\pi R}{1 - R/2h}$	meter	[5.16, 5.17]

Table 5.4 Conduction shape factors: $Q = S\,k\Delta T$, $R_t = 1/(kS)$ *(con't).*

<table>
<tr><th>Situation</th><th>Shape factor, S</th><th>Dimensions</th><th>Source</th></tr>
<tr><td>8. An isothermal sphere of radius R, near an insulated plane, transfers heat to a semi-infinite medium at T_∞ (see items 4 and 7)



</td><td>$$\frac{4\pi R}{1 + R/2h}$$</td><td>meter</td><td>[5.18]</td></tr>
<tr><td>9. Parallel cylinders exchange heat in an infinite conducting medium



</td><td>$$\frac{2\pi}{\cosh^{-1}\left(\dfrac{L^2 - R_1^2 - R_2^2}{2R_1R_2}\right)}$$</td><td>none</td><td>[5.6]</td></tr>
<tr><td>10. Same as 9, but with cylinders widely spaced; $L \gg R_1$ and R_2</td><td>$$\frac{2\pi}{\cosh^{-1}\left(\dfrac{L}{2R_1}\right) + \cosh^{-1}\left(\dfrac{L}{2R_2}\right)}$$</td><td>none</td><td>[5.16]</td></tr>
<tr><td>11. Cylinder of radius R_i surrounded by eccentric cylinder of radius $R_o > R_i$; centerlines a distance L apart (see item 2)</td><td>$$\frac{2\pi}{\cosh^{-1}\left(\dfrac{R_o^2 + R_i^2 - L^2}{2R_oR_i}\right)}$$</td><td>none</td><td>[5.6]</td></tr>
<tr><td>12. Isothermal disc of radius R on an otherwise insulated plane conducts heat into a semi-infinite medium at T_∞ below it</td><td>$4R$</td><td>meter</td><td>[5.6]</td></tr>
<tr><td>13. Isothermal ellipsoid of semimajor axis b and semiminor axes a conducts heat into an infinite medium at T_∞; $b > a$ (see 4)</td><td>$$\frac{4\pi b\sqrt{1 - a^2/b^2}}{\tanh^{-1}\left(\sqrt{1 - a^2/b^2}\right)}$$</td><td>meter</td><td>[5.16]</td></tr>
</table>

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