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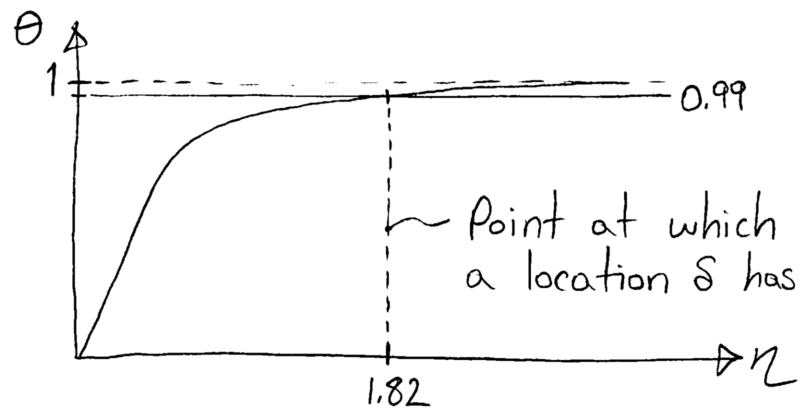
$$q''\Big|_{x=0} = -k(T_i - T_s)\frac{\partial\theta}{\partial x}\Big|_{x=0} = k\underbrace{(T_s - T_i)}_{\Delta T}\underbrace{\frac{\partial\eta}{\partial x}}_{\frac{1}{2\sqrt{\alpha t}}}\cdot\underbrace{\frac{\partial\theta}{\partial\eta}\Big|_{x=0}}_{\frac{2}{\sqrt{\pi}}}$$

Remember: $\frac{\partial\theta}{\partial\eta} = \theta' = C_2\, e^{-\eta^2} = \frac{2}{\sqrt{\pi}}e^{-\eta^2}$

$$\boxed{q'' = \frac{k\Delta T}{\sqrt{\pi\alpha t}}}\ , \quad \boxed{\alpha = \frac{k}{\rho c}}$$ $\Rightarrow$ Thermal Diffusivity [Material prop.]

Note, to get total energy transfered [J], you need to integrate q'' with respect to t.

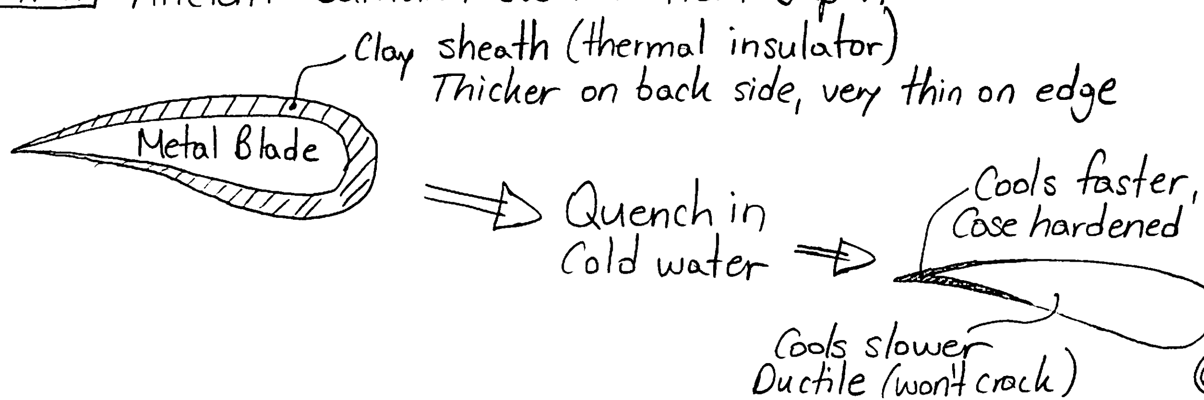
One last thing:



Point at which the body temperature at a location δ has reached a temperature that is 99% of the difference between T_s & T_i. $\theta = 0.99$

$$\frac{\delta}{2\sqrt{\alpha t}} = 1.82 \Rightarrow \boxed{\delta(t) = 3.64\sqrt{\alpha t}}$$ $\Rightarrow$ Thermal penetration depth.

Application | Ancient samurai swords from Japan



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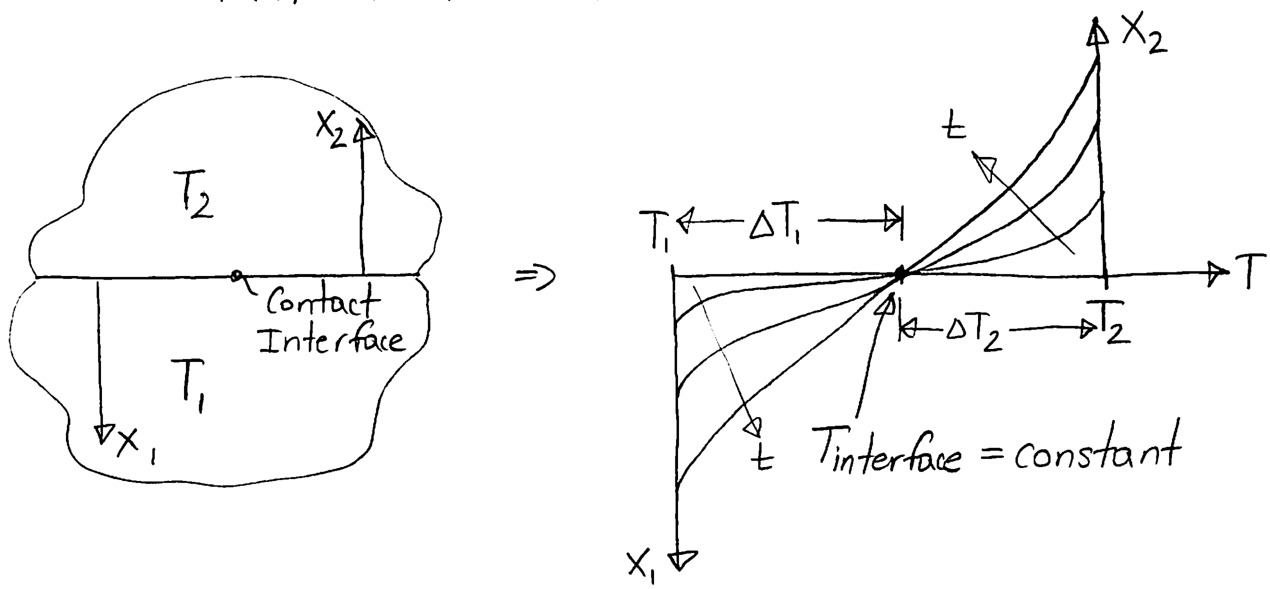
Contact Between Two Semi-Infinite Solids

When two bodies at different initial temperatures are brought into contact, they initially achieve a temperature that is constant at their interface:



How do we make sure this is indeed true?
Let's apply an energy balance on the interface:

$$q'' = \frac{k\Delta T}{\sqrt{\pi \alpha t}} = \frac{(k\rho c)^{1/2}}{\sqrt{\pi}} \cdot \frac{\Delta T}{\sqrt{t}}$$

$$q''_1 = q''_2 = \frac{(k_1\rho_1 c_1)^{1/2}}{\cancel{\sqrt{\pi}}} \cdot \frac{\Delta T_1}{\cancel{\sqrt{t}}} = \frac{(k_2\rho_2 c_2)^{1/2}}{\cancel{\sqrt{\pi}}} \cdot \frac{\Delta T_2}{\cancel{\sqrt{t}}}$$

$$(k_1\rho_1 c_1)^{1/2}\Delta T_1 = (k_2\rho_2 c_2)^{1/2}\Delta T_2 \quad (1)$$

But we also know that: $\Delta T_1 + \Delta T_2 = \Delta T$ (2)
Back substituting (2) into (1):

$$\Delta T_2 = \frac{(k\rho c)_1^{1/2}}{(k\rho c)_2^{1/2}}\Delta T_1 \quad \Rightarrow \quad \Delta T = \left(1 + \sqrt{\frac{(k\rho c)_1}{(k\rho c)_2}}\right)\Delta T_1$$

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$$\boxed{\Delta T_1 = \frac{\Delta T}{1+\sqrt{\frac{(k\rho c)_1}{(k\rho c)_2}}} = \frac{\sqrt{(k\rho c)_2}\cdot \Delta T}{\sqrt{(k\rho c)_2}+\sqrt{(k\rho c)_1}} \neq f(t)}$$ ⇒ not a function of time!

$$\boxed{q\big|_{x=0} = \frac{\sqrt{(k\rho c)_1}\sqrt{(k\rho c)_2}}{\sqrt{(k\rho c)_1}+\sqrt{(k\rho c)_2}}\cdot\frac{\Delta T}{\sqrt{\pi t}}}$$ ⇒ Heat flux between two contacting semi-infinite bodies.

This is why when you touch certain objects at room temperature, they feel "colder" than others, even though they are at the same temperature.

Example | Brass & Wooden Doorknobs

Brass: $k_1 = 109\ W/m\cdot K$, $\rho_1 = 8730\ kg/m^3$, $C_1 = 380\ J/kg\cdot K$

$$(k_1\rho_1C_1)^{1/2} = 19016\ J/m^2Ks^{1/2}$$

Wood: $k_2 = 0.17\ W/m\cdot K$, $\rho_2 = 750\ kg/m^3$, $C_2 = 1700\ J/kg\cdot K$

$$(k_2\rho_2C_2)^{1/2} = 466\ J/m^2Ks^{1/2}$$

Your body temperature is $\approx 37°C$
Room temperature of a cold room $\approx 17°C$

$$\Delta T = T_{body} - T_{Room} = 20°C$$

Now we need human flesh properties:

Human hand: $k_3 = 0.6\ W/m\cdot K$ (Water), $\rho_3 \approx 1000\ kg/m^3$, $C_3 = 4190\ J/kg\cdot K$

$$(k_3\rho_3C_3)^{1/2} = 1586\ J/m^2Ks^{1/2}$$

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Case 1: You reach for the brass doorknob:

$$q''_{3,1} = \frac{(19016)(1586)}{(19016)+(1586)}\frac{\Delta T}{\sqrt{\pi t}} = 1463\frac{\Delta T}{\sqrt{\pi t}}$$

⇒ Heat transfer from your hand to the brass doorknob.

Case 2: You reach for the wooden doorknob:

$$q''_{3,2} = \frac{(466)(1586)}{(466)+(1586)}\cdot\frac{\Delta T}{\sqrt{\pi t}} = 360\frac{\Delta T}{\sqrt{\pi t}}$$

⇒ Heat transfer from your hand to the wooden doorknob.

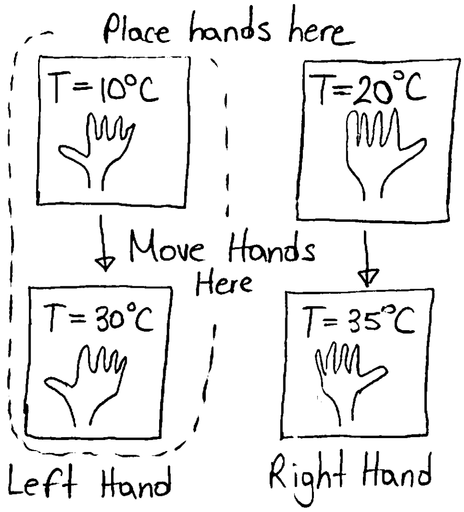
Comparing, we see that:

$$\boxed{\frac{q''_{3,1}}{q''_{3,2}} = 4.1}$$

⇒ This is why the brass doorknob feels so much colder. It pulls out heat from your hand at a rate 4 times faster than the wood.

On another note, your body translates temperature or feeling hot or cold by sensing heat flow, not absolute temperature. The faster you lose heat, the colder something feels.

Try the following:



Your left hand (dotted circle) will feel much hotter than your right, even though the second right plate is at a higher temperature!