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Radiation: ⇒ Heat transfer via electromagnetic waves
No need for a medium ⇒ not diffusion

Examples: 1) Sun heating the earth
2) Radiative heaters in store entrances & theaters
3) IR (infrared) cameras & detection

Fourier's Law of Heat Conduction

In 17'th century, a guy named Joseph Fourier discovered an emperical correlation that governs heat transfer in a solid:

$$\boxed{q''_x = -k\frac{dT}{dx}}$$

Heat flux in the x direction

Thermal conductivity of the material transfering heat

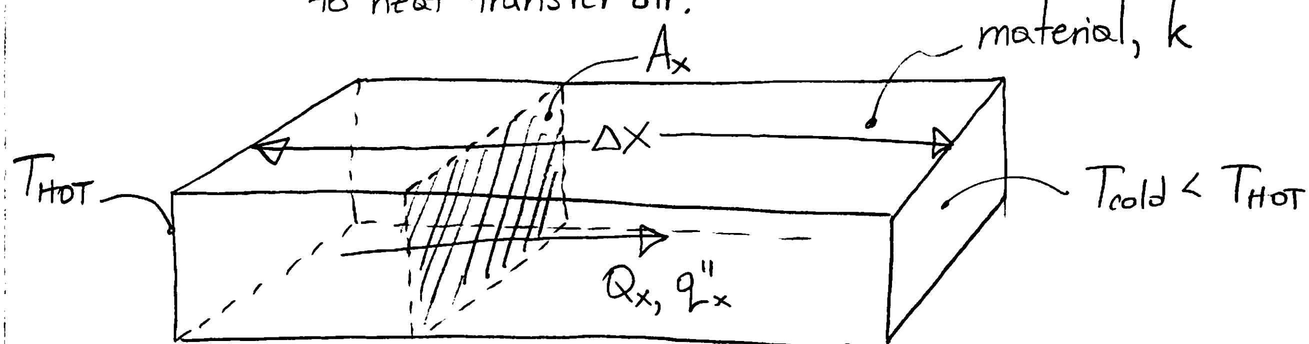
Temperature gradient in the x-direction. Ordinary derivative

Sometimes written as:

$$\boxed{Q_x = -kA_x\frac{dT}{dx}}$$

Cross sectional area perpendicular to heat transfer dir.

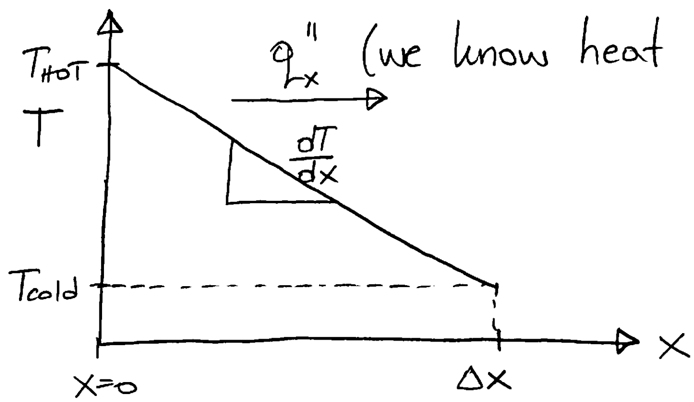
$$\frac{dT}{dx} = \frac{T_{cold} - T_{HOT}}{x}$$



Fourier did this experiment for many metals to obtain his relation.

Let's take a closer look at his equation:

$$q''_x = -k\frac{dT}{dx}$$

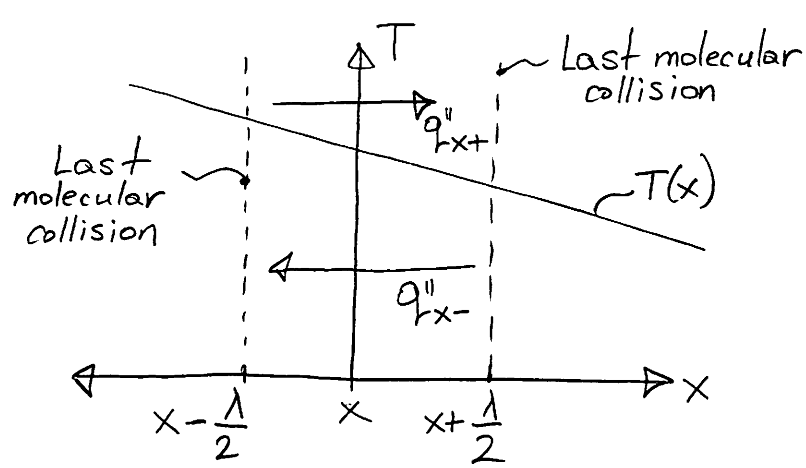


$$\frac{dT}{dx} = \frac{T_{cold} - T_{HOT}}{\Delta x} < 0 \quad \text{(negative)}$$

But q''_x must be positive since heat flows in the positive x-direction?

→ Hence the negative sign in the equation. k is always a positive quantity, hence things make sense. Heat flows down the temperature gradient.

Derivation of Fourier's Law



$n \equiv$ # of particles/m^3
$m \equiv$ mass per particle
$C_p \equiv$ specific heat capacity
$\bar{c} \equiv$ average speed of partick

$C_p\ [J/kg\cdot K]$; $n\ [\#/m^3]$, $m\ [\frac{kg}{\#}$

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Lets look at the energy exchange at our plane of interest (x)

$$q''_{x+} = n \cdot m \cdot C_p \cdot \bar{c} \cdot \underbrace{T\left(x-\frac{\lambda}{2}\right)}_{\text{Temperature at } x = x - \frac{\lambda}{2}}$$

$$\underbrace{[\#/m^3] \cdot [kg/\#] \cdot [m/s]}_{\left[\frac{kg}{m^2 \cdot s}\right]} \cdot \underbrace{[J/kg \cdot K] \cdot [K]}_{[J/kg]}$$

$\left[\frac{kg}{m^2 \cdot s}\right]$ mass flow rate of particles per unit area.

$[J/kg]$ energy flow per unit mass of particles.

We can notice that: $n \cdot m = \rho \; [kg/m^3] \Rightarrow$ Density

$$q''_{x+} = +\rho C_p \bar{c}\, T\left(x-\frac{\lambda}{2}\right)$$

$$q''_{x-} = +\rho C_p \bar{c}\, T\left(x+\frac{\lambda}{2}\right)$$

$$q''_{x} = q''_{x+} - q''_{x-} = \rho C_p \bar{c}\left[T\left(x-\frac{\lambda}{2}\right) - T\left(x+\frac{\lambda}{2}\right)\right]$$

$$= -\rho C_p \bar{c}\left[T\left(x+\frac{\lambda}{2}\right) - T\left(x-\frac{\lambda}{2}\right)\right]$$

Mutiply the right hand side by (λ/λ):

$$q''_{x} = -\rho C_p \bar{c} \lambda \underbrace{\left[\frac{T\left(x+\frac{\lambda}{2}\right) - T\left(x-\frac{\lambda}{2}\right)}{\lambda}\right]}_{\frac{dT}{dx}}$$

Typical Values

$k_{air} \approx 0.1$ W/m·K

$k_{water} = 0.6$

$k_{polymer} \approx 0.1$

$k_{Cu} = 400$

$k_{Al} = 300$

$$\boxed{q''_{x} = -\underbrace{\rho C_p \bar{c} \lambda}\frac{dT}{dx}}$$

$$k = \rho C_p \bar{c} \lambda \Rightarrow \left[\frac{\cancel{kg}}{m^{\cancel{3}}}\right] \cdot \left[\frac{J}{\cancel{kg} \cdot K}\right]\left[\frac{\cancel{m}}{s}\right][\cancel{m}] = \frac{\overset{W}{J}}{m \cdot s \cdot K} = \left[\frac{W}{m \cdot K}\right]$$

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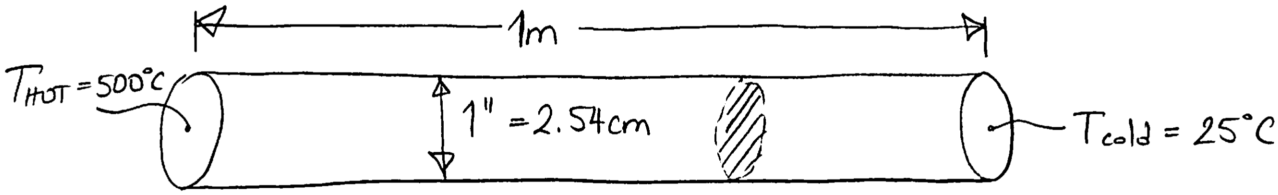
Example | Conduction through a bar:
A bar of copper ($k_{cu} = 400\ W/m\cdot K$) contacts a hot source at 500°C on one end. The opposite end of the bar is cooled to ambient ($T_{cold} = 25°C$) by cooling water. The length of the bar is 1m, and it is cylindrical with a diameter of 1". What is the heat transfer rate? What is the heat flux?



$$q'' = -k\frac{dT}{dx} = -(400\ W/m\cdot K)\cdot\left(\frac{25°C - 500°C}{1m}\right)$$

$$= 190\,000\ W/m^2$$

$$\boxed{q'' = 19\ W/cm^2}$$

$$Q = -kA\frac{dT}{dx} = q''A = (19\ W/cm^2)\left(\pi\frac{(2.54cm)^2}{4}\right)$$

$$\boxed{Q = 96.2\ W}$$

So how large is this?

$q''_{boiling} \approx 100\ W/cm^2$

$q''_{nat.\ convection} \approx 0.01\ W/cm^2$

Think about power plants: $\dot{W} = 1000\ MW$ electrical output
Assuming efficiency of 50% ($\eta = 0.5$)

$$\eta = \frac{\dot{W}}{Q_{in}} \Rightarrow \boxed{Q_{in} = 2\,000\ MW = 2\,000\,000\,000\ W!}$$

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The previous example was a good demonstration of Fourier's law, but it cannot be used for more complex situations:

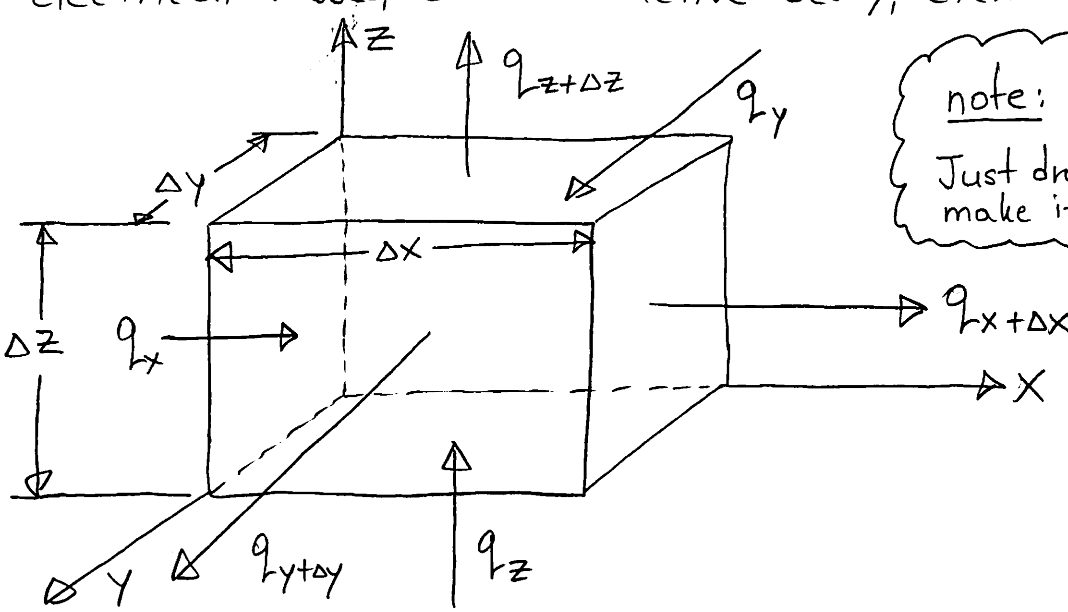
- Transient heat transfer
- Heat loss to the air along the length of the bar
- Radiation effects
- Heat generation.

We need a more general formulism to handle these problems.

<u>Conduction Heat Transfer</u> (Chapter 2 of Textbook)

The equation of conduction heat transfer (heat diffusion eqn.) is the first law of thermodynamics subject to the following assumptions:

1) No mass flow in or out of the control volume (C$\forall$) or system.
2) No change in Kinetic or Potential energy
3) There may exist a heat generation term, eg. I^2R from electrical losses, or radioactive decay, etc...



note: $q = q''$ [W/m²]
Just dropped the " to make it easier.