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This is awesome because now we can calculate all kinds of interesting & usefull things:

Shear Stress:

$$\tau(x) = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \rho \upsilon V_\infty \frac{\partial \bar{u}}{\partial y}\bigg|_{y=0}$$

$$= \rho \upsilon V_\infty \frac{\partial \bar{u}}{\partial \eta}\bigg|_{\eta=0} \cdot \frac{\partial \eta}{\partial y}\bigg|_{y=0}$$

$$\tau(x) = \rho \upsilon V_\infty a_2 \sqrt{\frac{V_\infty}{x\upsilon}}$$

Typically, you're used to seeing it in terms of a skin friction coefficient, C_D

Reynolds number

$$\boxed{C_{D,x} = \frac{\tau(x)}{\frac{1}{2}\rho V_\infty^2} = \frac{2a_2}{\sqrt{Re_x}} = \frac{0.664}{Re_x^{1/2}}} \quad , \quad \boxed{Re_x = \frac{\rho V_\infty x}{\mu}}$$

↳ Skin friction coefficient for a flat plate in laminar flow.

Typically we want the average shear:

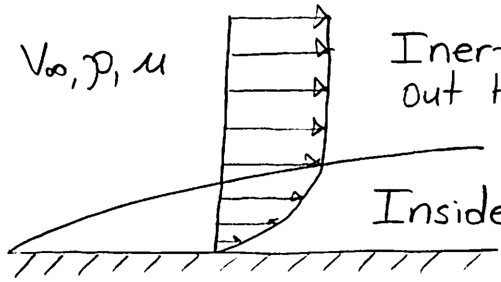
$$\bar{\tau} = \frac{1}{L}\int_0^L \tau(x)\,dx \Rightarrow \tau(x) = C \cdot \frac{1}{\sqrt{x}}$$

$$= C \cdot \frac{1}{L}\int_0^L \frac{dx}{\sqrt{x}} = \frac{2C}{\sqrt{L}}$$

$$\boxed{C_D = \frac{\bar{\tau}}{\frac{1}{2}\rho V_\infty^2} = \frac{1.328}{Re_L^{1/2}}} \quad , \quad \boxed{Re_L = \frac{\rho V_\infty L}{\mu}}$$

↳ Plate averaged skin friction coefficient

Note, there is a much easier way to solve for all this. Use <u>scaling</u>:



Inertia Dominates out Here $(y > \delta)$ $\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x}$ (Inviscid flow)

Inside the b.l. $(y < \delta)$, inertia balances with viscous forces. Viscous effects become very important.

$$\text{Inertia} \sim \text{Viscosity}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \sim \nu\frac{\partial^2 u}{\partial y^2}$$

$$V_\infty \frac{V_\infty}{L} + V_\infty\left(\frac{\cancel{\delta}}{L}\right)\cdot\frac{V_\infty}{\cancel{\delta}} \sim \nu\frac{V_\infty}{\delta^2}$$

$$\frac{V_\infty^{\cancel{2}}}{L} \sim \nu\frac{\cancel{V_\infty}}{\delta^2}$$

$$\delta^2 \sim \frac{\mu L}{\rho V_\infty} \Rightarrow \delta \sim \sqrt{\frac{\nu L}{V_\infty}}\cdot\left(\frac{L}{L}\right) \Rightarrow \boxed{\delta \sim \frac{L}{\sqrt{Re_L}}}$$ OMG! So easy!

Note, this scaling result is great for giving you a sense of orders of magnitude, however not as accurate as analytical solutions.

Example | Calculate the boundary layer thickness on a 737 jet.

$V_\infty = 400$ miles/hour $= 177$ m/s (Plane speed)
$\nu_{air} = 1.5 \times 10^{-5}$ m²/s
$L = 5$m (Length of the wing in the fuselage direction)

$$Re_L = \frac{V_\infty L}{\nu} = 5.9 \times 10^7 \Rightarrow \boxed{\delta = \frac{5L}{Re_L} = 3.25\text{mm}}$$ $\Rightarrow$ Less than 1cm thick!

Very difficult to observe.