This is awesome because now we can calculate all kinds of interesting & useful things:

Shear Stress:

\[ \tau(x) = \mu \frac{\partial u}{\partial y} \Bigg|_{y=0} = \rho u v_{\infty} \frac{\partial u}{\partial y} \Bigg|_{y=0} \]
\[ = \rho u v_{\infty} \frac{\partial u}{\partial n} \bigg|_{n=0} \cdot \frac{\partial n}{\partial y} \bigg|_{y=0} \]

\[ \tau(x) = \rho u v_{\infty} a_2 \frac{v_{\infty}}{x U} \]

Typically, you're used to seeing it in terms of a skin friction coefficient, \( C_D \)

\[ C_{D,\text{avg}} = \frac{C(x)}{\frac{1}{2} \rho v_{\infty}^2} = \frac{2 a_2}{\sqrt{Re}} = \frac{0.664}{Re^{1/2}} \]

Typically we want the average shear:

\[ \bar{\tau} = \frac{1}{L} \int_0^L \tau(x) \, dx \Rightarrow \tau(x) = C \cdot \frac{1}{\sqrt{x}} \]

\[ = C \cdot \frac{1}{L} \int_0^L \frac{dx}{\sqrt{x}} = \frac{2C}{\sqrt{L}} \]

\[ C_D = \frac{\bar{\tau}}{\frac{1}{2} \rho v_{\infty}^2} = \frac{1.328}{Re^{1/2}} \]

E0) Plate averaged skin friction coefficient
Note, there is a much easier way to solve for all this. Use scaling:

\[ V_\infty, p, \mu \]

**Inertia Dominates** out here \((y > \delta)\) \(\Rightarrow u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \) (Inviscid flow)

**Inside the b.l.** \((y < \delta)\), inertia balances with viscous forces. Viscous effects become very important.

**Inertia \sim Viscosity**

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \sim U \frac{\partial^2 u}{\partial y^2} \]

\[ V_\infty \frac{V_\infty}{L} + V_\infty \left(\frac{S}{L}\right) \frac{V_\infty}{S} \sim U \frac{V_\infty}{S^2} \]

\[ \frac{V_\infty^2}{L} \sim U \frac{V_\infty}{S^2} \]

\[ S^2 \sim \frac{UL}{\rho V_\infty} \Rightarrow S \sim \sqrt{\frac{UL}{\rho V_\infty}} \cdot \left(\frac{L}{L}\right) \Rightarrow S \sim \frac{L}{\sqrt{Re}} \]

OMG! So easy!

Note, this scaling result is great for giving you a sense of orders of magnitude, however not as accurate as analytical solutions.

**Example** | Calculate the boundary layer thickness on a 737 jet.

\[ V_\infty = 400 \text{ miles/hour} = 177 \text{ m/s} \quad (\text{Plane speed}) \]

\[ U_{\text{air}} = 1.5 \times 10^{-5} \text{ m}^3/\text{s} \]

\[ L = 5 \text{ m} \quad (\text{Length of the wing in the fuselage direction}) \]

\[ Re_L = \frac{V_\infty L}{U} = 5.9 \times 10^7 \quad \Rightarrow \frac{S}{\sqrt{Re_L}} = 3.25 \text{ mm} \quad \Rightarrow \text{Less than 1mm thick!} \]

Very difficult to observe.