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Some observations & notes

All of our previous results are valid for the following conditions:

1) Re_x or $Re_L < 5.0\times10^5$ (Laminar flow)

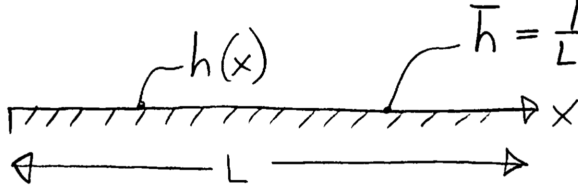
2) $Ma = \dfrac{V_\infty}{\text{speed of sound}} < 0.3$ (Incompressible flow)

3) $Ec = \dfrac{V_\infty}{C_p(T_s - T_\infty)} \ll 1$ (Viscous dissipation heating is negligible)
   $Ec \equiv$ Eckert #

4) We have assumed constant properties. Evaluate all properties (ρ, μ, Pr, etc...) at the film temp. (T_f)

$$T_f = \frac{T_s + T_\infty}{2} \Rightarrow \text{Use Table A.4 for gases, A.5 for liquids}$$

5) We have been working mostly with h in the class. Many times, its easier to work with averaged quantities



$\bar{h} = \frac{1}{L}\int_0^L h\,dx$ (Average heat transfer coefficient)

Also when we write: $Nu_x = \dfrac{h x}{k_f}$ (Local Nusselt number) ← h at x

$\overline{Nu_L} = \dfrac{\bar{h} L}{k_f}$ (Averaged Nusselt number)

6) $h \propto \dfrac{1}{\sqrt{x}}$ or $\bar{h} \propto \dfrac{1}{\sqrt{L}}$, $\Rightarrow Nu_x \propto \sqrt{x}$

Thus $h \to \infty$ and $Nu_x \to 0$ as $x \to 0$. Of course $h \to \infty$ does not really occur in reality, since the b.l. model breaks down at the leading edge ($x=0$).

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7) Different Boundary Conditions

So far, we've only been dealing with T_s = constant
What if we have uniform wall heat flux:

$$\underbrace{\bar{h} = \frac{\overline{q''}}{\Delta T}}_{T_s = \text{constant}} \qquad \underbrace{\bar{h} = \frac{q''}{\overline{\Delta T}}}_{q''|_{y=0} = \text{constant}}$$

Uniform Wall Temperature (T_s)

$$\bar{h} = \frac{\overline{q''}}{\Delta T} = \frac{1}{\Delta T}\left[\frac{1}{L}\int_0^L q''\,dx\right] = \frac{1}{L}\int_0^L h(x)\,dx$$

h as a function of x, not h times x.

Uniform Wall Heat Flux (q'')

$$\bar{h} = \frac{q''}{\overline{\Delta T}} = \frac{q''}{\frac{1}{L}\int_0^L \underbrace{\Delta T(x)}\,dx}$$

ΔT as a function of x, not ΔT times x

Nusselt Number Rules:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} \neq \frac{1}{L}\int_0^L Nu(x)\,dx$$

For a flat plate with T_s = constant

$$\bar{h} = \frac{1}{L}\int_0^L \underbrace{h(x)}_{\frac{k}{x}Nu_x}\,dx = \frac{0.332\,k\,Pr^{1/3}}{L}\cdot\sqrt{\frac{V_\infty}{\nu}}\int_0^L \frac{\sqrt{x}}{x}\,dx$$

$$\boxed{\bar{h} = 0.664\left(\frac{k}{L}\right) Re_L^{1/2}\, Pr^{1/3}}$$

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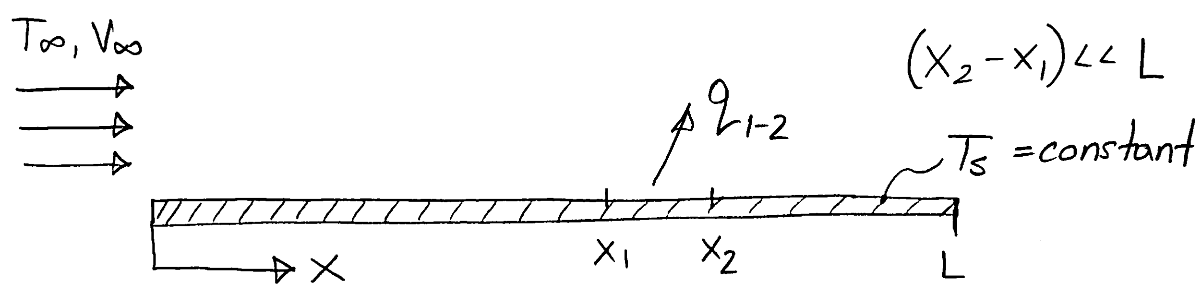
Example | Parallel flow over a flat plate. We need to analyze a short span of interest:



Find three different expressions for q_{1-2} (heat transfer)

① The best method:

$$q_{1-2} = \bar{h}_{1-2}(x_2-x_1)(T_s - T_\infty)$$

Solve for the local heat transfer coefficient since $(x_2-x_1) << L$

$$\boxed{\bar{h}_{1-2} \cong h_{\bar{x}} = h\left(x = \frac{x_2-x_1}{2}\right)}$$ ⇒ Best solution for this problem

② Approximation #1: Local coefficients

$$\boxed{\bar{h}_{1-2} \cong [h_{x_1} + h_{x_2}]/2}$$

③ Approximation #2: Average coefficients for x_1 & x_2

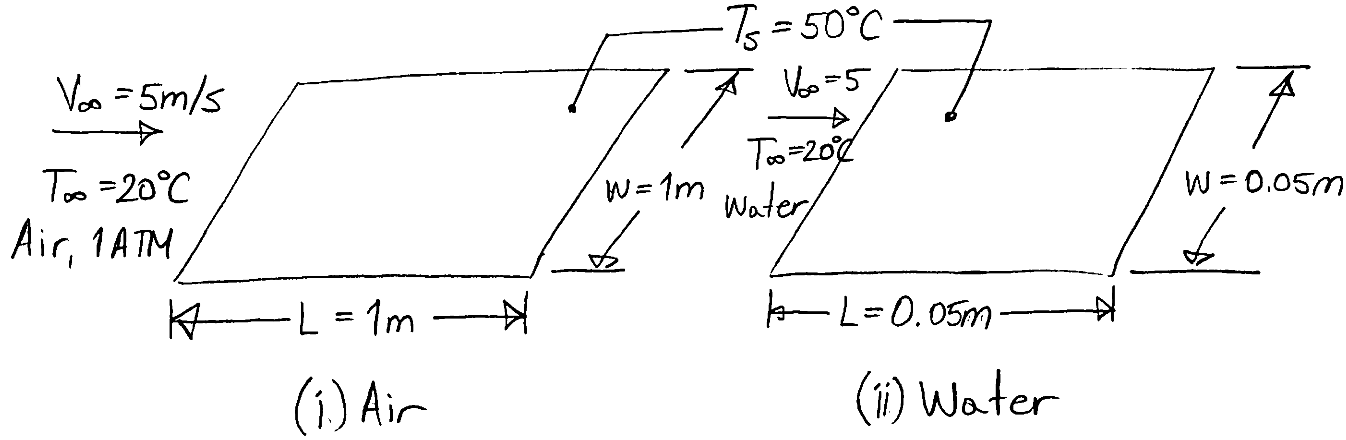
$$q_{1-2} = q_{0-2} - q_{0-1}$$

$$\bar{h}_{1-2}(x_2-x_1) = \bar{h}_2 x_2 - \bar{h}_1 x_1$$

$$\boxed{\bar{h}_{1-2} = \bar{h}_2 \frac{x_2}{x_2-x_1} - \bar{h}_1 \frac{x_1}{x_2-x_1}}$$ where $\bar{h}_2 = \frac{1}{x_2}\int_0^{x_2} h\,dx$

$\bar{h}_1 = \frac{1}{x_1}\int_0^{x_1} h\,dx$

Example | Air and Water flow over 2 flat plates:



Determine: (a) Average convective heat transfer coefficient
(b) Convective heat transfer rate
(c) Drag force on the plate

Assumptions: 1) Steady state
2) B.L. assumptions are valid
3) Constant properties

(a) Let's begin by computing our Reynolds number for each case:

$$Re_{L,Air} = \frac{V_\infty L}{\nu_{air}} = \frac{(5m/s)(1m)}{1.669\times10^{-5} m^2/s} = 3\times10^5 < 5\times10^5 \text{ (Laminar)}$$

$\hookrightarrow$ evaluated at $T_f = \frac{50°C + 20°C}{2} = 35°C$

$$Re_{L,water} = \frac{V_\infty L}{\nu_{water}} = \frac{(5m/s)(0.05m)}{7.291\times10^{-7} m^2/s} = 3.43\times10^5 < 5\times10^5 \text{ (L)}$$

So both flows are laminar, therefore we can use our previous solutions.

$$\overline{Nu_L} = \frac{\bar{h}L}{k_{fluid}} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

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Using Tables A.4 & A.5 in the Textbook to look up Pr

$Pr_{Air} = 0.706$ $\quad$ $Pr_{water} = 4.85$

$k_{Air} = 26.9 \times 10^{-3}\ W/m\cdot K$ $\quad$ $k_{water} = 0.625\ W/m\cdot K$

$\rho_{Air} = 1.135\ kg/m^3$ $\quad$ $\rho_{water} = 994\ kg/m^3$

$$\overline{Nu}_{L,Air} = \frac{\bar{h}_{Air}(1m)}{(26.9\times10^{-3} W/m\cdot K)} = 0.664\left(3\times10^5\right)^{1/2}(0.706)^{1/3}$$

$$\boxed{\bar{h}_{Air} = 8.71\ W/m^2\cdot K}$$

$$\overline{Nu}_{L,water} = \frac{\bar{h}_{water}(0.05m)}{(0.625\ W/m\cdot K)} = 0.664\left(3.43\times10^5\right)^{1/2}(4.85)^{1/3}$$

$$\boxed{\bar{h}_{WATER} = 8228.21\ W/m^2\cdot K}$$

Note the Yuge! difference.
The smaller boundary layer length for water jacks up the $\bar{h}$.

(b) $$q_{Air} = \bar{h}_{Air}\cdot w\cdot L\cdot(T_s - T_\infty)$$

$$= (8.71\ W/m^2\cdot K)(1m)(1m)(50°C - 20°C)$$

$$\boxed{q_{Air} = 174.2\ W}$$

$$q_{water} = \bar{h}_{WATER}\cdot w\cdot L\cdot(T_s - T_\infty)$$

$$= (8228.21\ W/m^2\cdot K)(0.05m)(0.05m)(50°C - 20°C)$$

$$\boxed{q_{WATER} = 617.1\ W}$$

Even though the water plate area is 400X smaller than the air plate, the heat transfer is 3.5X larger!

This is why when you fall in a frozen lake at 0°C you get hypothermia in minutes, but chilling outside at 0°C is ok for hours.

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(c) Now we can solve for shear force (drag)

$$\overline{C_{D,Air}} = \frac{\overline{\tau}}{\frac{1}{2}\rho V_\infty^2} = \frac{1.328}{Re_L^{1/2}} \quad \text{(pg. 110 of notes)}$$

$$= \frac{1.328}{(3\times10^5)^{1/2}} = 0.00242$$

$$F_{Drag,Air} = \overline{\tau}\cdot w\cdot L = \overline{C_{D,Air}}\,\tfrac{1}{2}\rho_{Air} V_\infty^2\, w\cdot L$$

$$= (0.00242)\left(\tfrac{1}{2}\right)(1.135\,kg/m^3)(5\,m/s)^2(1\,m^2)$$

$$\boxed{F_{Drag,Air} = 0.034\,N}$$

$$F_{Drag,WATER} = \overline{\tau}\cdot w\cdot L = \overline{C_{D,WATER}}\cdot\tfrac{1}{2}\rho_{WATER}\cdot V_\infty^2\cdot w\cdot L$$

$$= (0.0023)\left(\tfrac{1}{2}\right)(998\,kg/m^3)(5\,m/s)^2(0.05\,m)(0.05\,m)$$

$$\boxed{F_{Drag,WATER} = 0.072}$$

So even though the water plate area is 400X smaller, the drag force on the plate is 2x larger than the air case!

This example shows the tradeoff of using air & water.

$$\overline{h}_{WATER} \gg \overline{h}_{Air}$$

but

Pressure drop in the flow

$$F_{Drag,WATER} \gg F_{Drag,air} \quad \text{or} \quad \Delta P_{WATER} \gg \Delta P_{air}$$

So you need way more energy to drive your flow for water.

EXTERNAL FLOW CORRELATIONS FOR CONVECTION

(For isothermal surfaces)

Table 7.7 in your edition, pg. 484 (Seventh Edition)

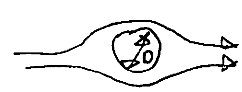
TABLE 7.9 Summary of convection heat transfer correlations for external flow[a, b]

Correlation		Geometry	Conditions[c]
$\delta = 5x\,Re_x^{-1/2}$	(7.19)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	(7.20)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2}\,Pr^{1/3}$	(7.23)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	(7.24)	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$	(7.29)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664 Re_x^{1/2}\,Pr^{1/3}$	(7.30)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565 Pe_x^{1/2}$	(7.32)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	(7.34)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\,Re_x^{-1/5}$	(7.35)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296 Re_x^{4/5}\,Pr^{1/3}$	(7.36)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	(7.40)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037 Re_L^{4/5} - 871)Pr^{1/3}$	(7.38)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$ (Table 7.2)	(7.52)	Cylinder	Average, T_f, $0.4 \lesssim Re_D \lesssim 4 \times 10^5$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C\,Re_D^m\,Pr^n(Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.53)	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$
$\overline{Nu}_D = 0.3 + [0.62 Re_D^{1/2}\,Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}] \times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(7.54)	Cylinder	Average, T_f, $Re_D\,Pr \gtrsim 0.2$
$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3})Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	(7.56)	Sphere	Average, T_∞, $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$
$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2}\,Pr^{1/3}$	(7.57)	Falling drop	Average, T_∞
$\overline{Nu}_D = 1.13 C_1 C_2\,Re_{D,\max}^m\,Pr^{1/3}$ (Tables 7.5, 7.6)	(7.60), (7.61)	Tube bank[d]	Average, $\overline{T}_f$, $2000 \lesssim Re_{D,\max} \lesssim 4 \times 10^4$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C C_2\,Re_{D,\max}^m\,Pr^{0.36}(Pr/Pr_s)^{1/4}$ (Tables 7.7, 7.8)	(7.64), (7.65)	Tube bank[d]	Average, $\overline{T}$, $1000 \lesssim Re_D \lesssim 2 \times 10^6$, $0.7 \lesssim Pr \lesssim 500$

some Definitions

$$Re_x = \frac{\rho V_\infty x}{\mu} \; ; \; Pr = \frac{\nu}{\alpha} \; ; \; Re_D = \frac{\rho V_\infty D}{\mu}$$

D ← Pipe outer diameter



$$Nu_x = \frac{hx}{k_f} \; ; \; \nu = \frac{\mu}{\rho} \; ; \; Nu_D = \frac{hD}{k_f} \; ; \; \overline{Nu}_D = \frac{\overline{h}D}{k_f}$$