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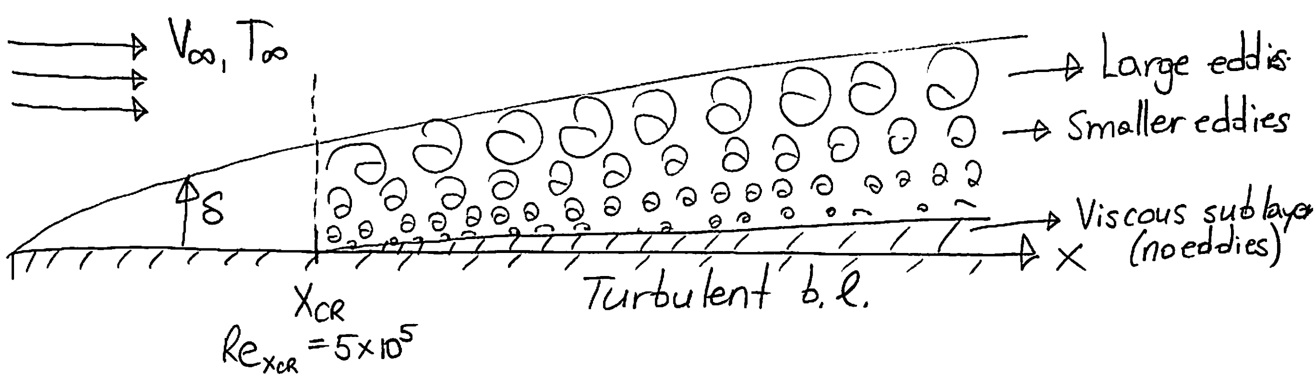
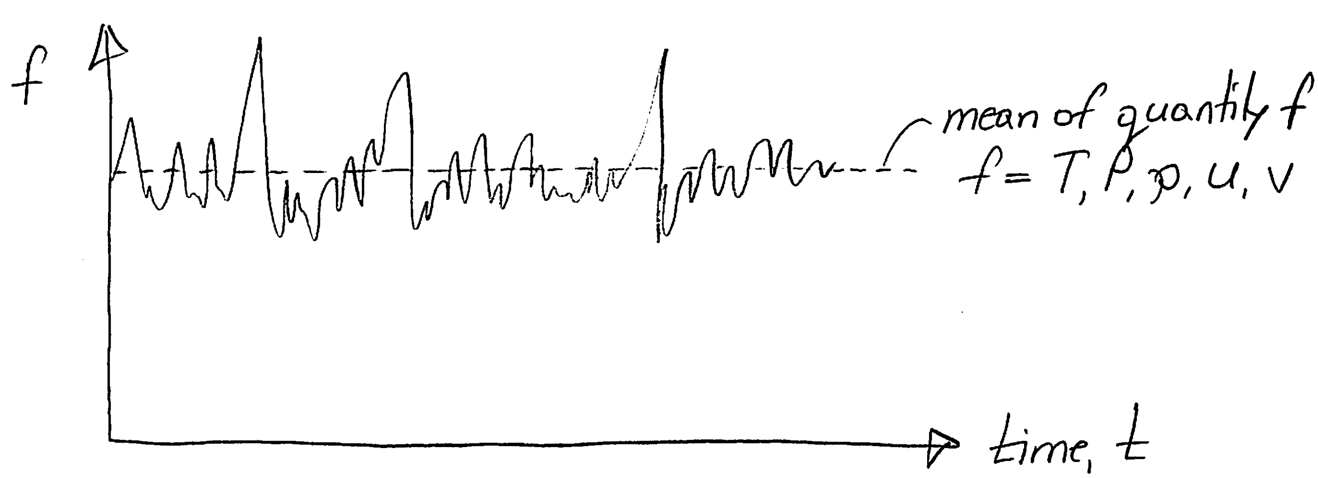
Turbulent Flow

Turbulence in a fluid can be seen as a spectrum of coexisting vortices (eddies) in which kinetic energy from larger ones is dissipated to succesively smaller ones until the very smallest of these vortices are damped out by viscous shear.

For a flat plate, external flow:



We handle this chaos by time averaging. Turbulent flow causes fluctuations of the velocity components, pressure, temperature, and in compressible flow, density.



By defining $f = \bar{f} + f'$, we can back substitute into our momentum equations & solve (ME 420)

$\bar{f}$ Mean Component, f' Fluctuating Component

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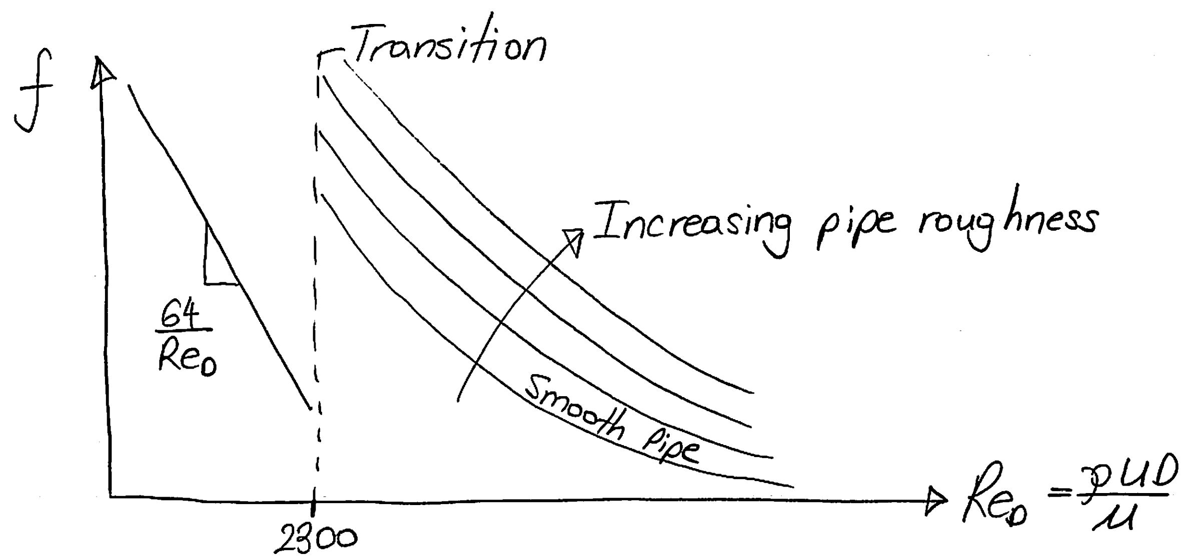
We handle the mathematics in ME420 & 521, for 320, we learn to use the results: (correlations)

One of the best & useful results is the emperical Moody chart:



Good correlation to know (if you have a smooth pipe)

$$\boxed{f = \left(0.79 \ln(Re_D) - 1.64\right)^{-2}} \quad 10^4 < Re_D < 10^6, \text{ smooth pipe}$$

$$\boxed{f = \frac{\Delta P}{\left(\frac{L}{D}\right)\frac{1}{2}\rho u^2}}, \quad \boxed{C_f = \frac{\overline{\tau}_o}{\frac{1}{2}\rho u^2}}, \quad \boxed{C_f = \frac{f}{4}} \text{ (for smooth pipes)}$$

L = pipe length, D = pipe diameter, u = average flow velocity
$\overline{\tau}_o$ = average shear stress at the pipe wall

You may ask, why friction & pressure drop, this isn't fluid mechanics. Well, momentum transfer & heat transfer are intricatly linked, hence to solve for heat transfer, we need fluid mechanics, as we will see in the Nusselt # correlations

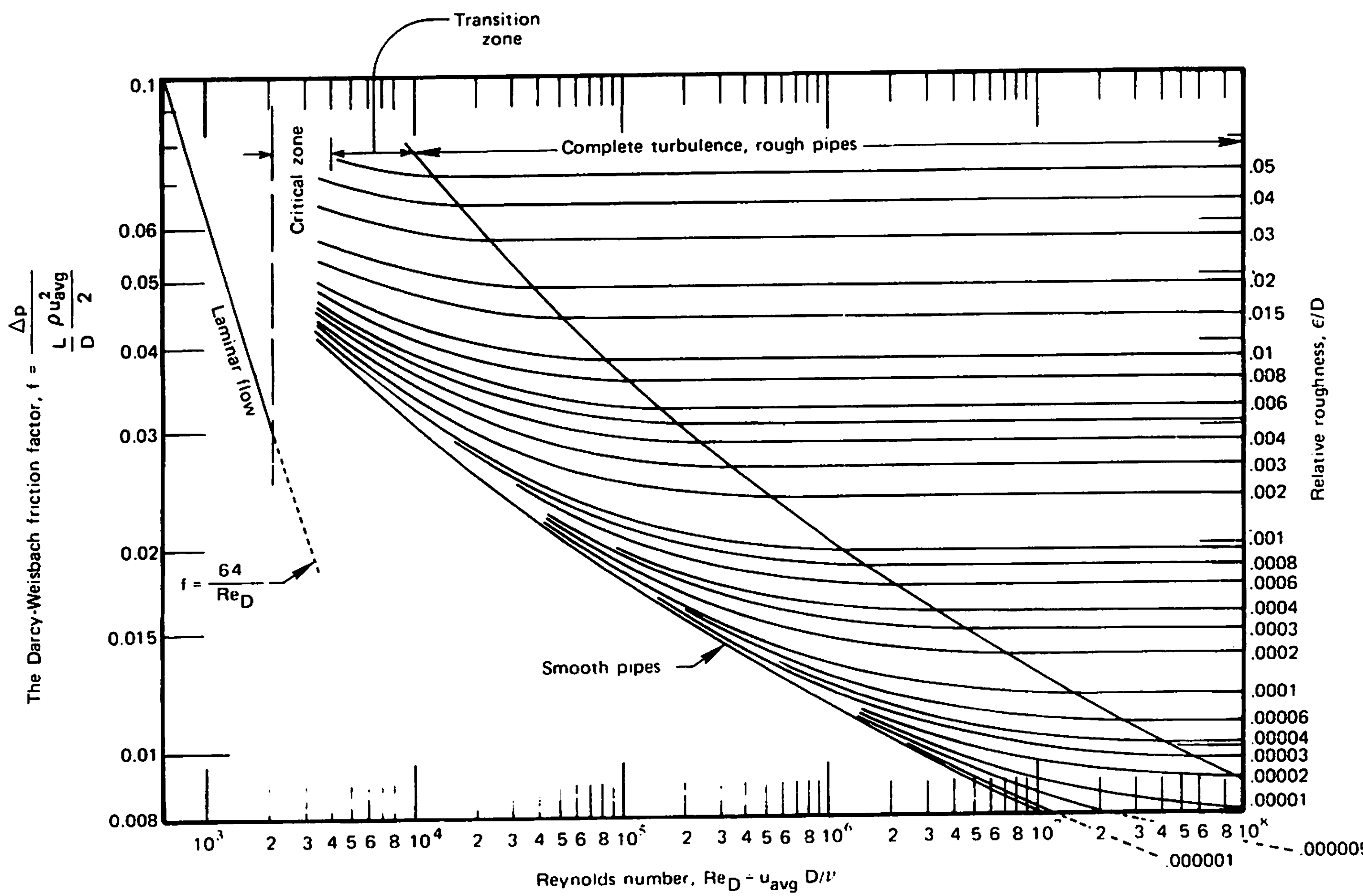


Figure 7.6 Pipe friction factors.

*Adapted from Lienhard & Lienhard, "A Heat Transfer Textbo

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Turbulent Flow Heat Transfer (Internal flow)

Some very good correlations to know are:

$$\boxed{\overline{Nu_D} = \frac{\bar{h} D}{k_f} = 0.023\, Re_D^{0.8} Pr^{0.4}}$$

⇒ Dittus - Boelter eq.

⇒ $Re_D > 10{,}000$

For a more accurate result, use:

$$\boxed{Nu_D = \frac{\bar{h} D}{k_f} = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7 (f/8)^{1/2} (Pr^{2/3} - 1)}}$$

⇒ Gnielinski correlati[on]

$3000 < Re_D < 5 \times 10^6$

External Flow (Turbulent heat transfer)

Remember, $Re_{CR} = \frac{V_\infty X_{CR}}{\nu} = 5 \times 10^5$ (Turbulent transition)

Some useful correlations: ($\tau_{o,x}$ = local shear stress at x)

$$\boxed{C_{f,x} = \frac{\tau_{o,x}}{\frac{1}{2} \rho V_\infty^2} = 0.0592\, Re_x^{-1/5}}$$

$10^5 < Re_x < 10^7$
Local skin friction coefficient at x.

$$\boxed{C_{f,x} = \frac{\tau_{o,x}}{\frac{1}{2} \rho V_\infty^2} = 0.026\, Re_x^{-1/7}}$$

$10^7 < Re_x < 10^9$

Note, these external flow correlations are also on pg. 123 of the notes (Table 7.7)

For even greater accuracy: $\boxed{C_{f,x} = \frac{0.455}{\ln(0.06\, Re_x)^2}}$ $10^5 < Re_x < 10^9$

↳ White correlation

Typically, we need averaged skin friction coefficient for the whole plate:

$$\tau_{0,x} = \frac{1}{2}\rho V_\infty^2 C_{f,x} = \frac{1}{2}\rho V_\infty^2 \cdot 0.0592\, Re_x^{-0.2}$$

$$\overline{\tau_{av,L}} = \frac{1}{L}\int_0^L \tau_{0,x}\, dx = \frac{\tau_{0,L}}{0.8}$$

$$C_{f,av} = \overline{C_f} = \frac{C_{f,L}}{0.8} = \frac{0.0592\, Re_L^{-1/5}}{0.8}$$

$$\boxed{\overline{C_f} = 0.074\, Re_L^{-1/5}}$$ ⇒ Plate averaged skin friction coefficient.

<u>External Heat Transfer</u>

$$\boxed{Nu_x = \frac{hx}{k_f} = 0.029\, Re_x^{0.8}\, Pr^{1/3}}$$ ⇒ Local Nusselt #
Analytical Result

$$\boxed{Nu_x = \frac{hx}{k_f} = 0.029\, Re_x^{0.8}\, Pr^{0.43}}$$ ⇒ Local Nusselt #
Exerimental Result
Whitacker Correlation

As we can see, the analytical & experimental results match pretty well.

$$\boxed{\overline{Nu_L} = \frac{\bar{h}L}{k_f} = 0.036\, Re_L^{0.8}\, Pr^{0.43}}$$

$0.7 < Pr < 400$
$5\times10^5 < Re_L < 3\times10^7$

For a more accurate correlation:

$$\boxed{Nu_x' = \frac{\left(\frac{C_{f,x}}{2}\right) Re_x\, Pr}{1 + 12.7\left(\frac{C_{f,x}}{x}\right)^{1/2}\left(Pr^{2/3} - 1\right)}}$$

⇒ White Correlation
$0.5 < Pr < 2000$
$5\times10^5 < Re_x < 10^7$