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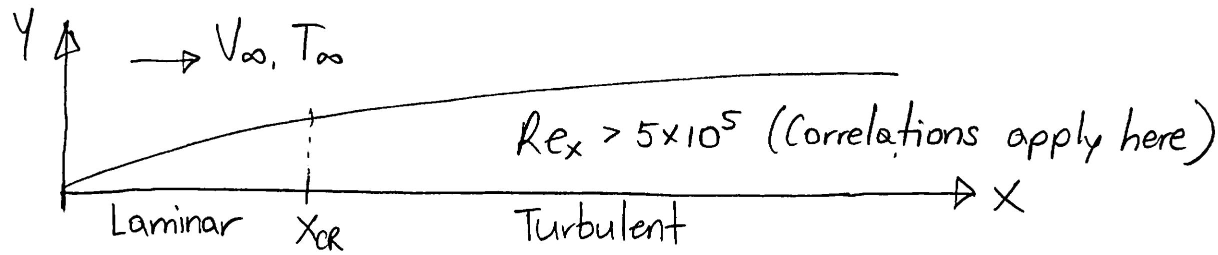
Note, so far, everything we've been covering has been in the turbulent region of the plate, but there is still a laminar section. How do we handle this?

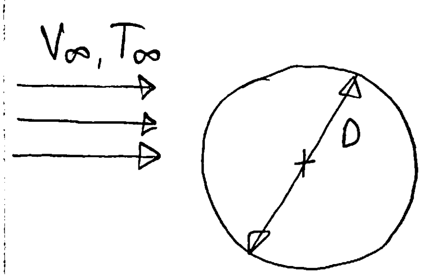


Mixed flow correlations exist (pg. 123 of notes)

$$\boxed{\overline{C_{f,L}} = 0.074\,Re_L^{-1/5} - 1742\,Re_L^{-1}} \quad Re_L < 10^8$$

$$\boxed{\overline{Nu_L} = \frac{\bar{h}L}{k_f} = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3}} \quad \begin{array}{l} Re_L < 10^8 \\ 0.6 < Pr < 60 \end{array}$$

<u>Cylinders & Spheres</u> (External flow)



$$\boxed{\overline{Nu_D} = \frac{\bar{h}D}{k_f} = \frac{1}{0.8237 - \ln(Re_D Pr)^{1/2}}}$$

↳ Cylinder, $Re_D Pr < 0.2$

$$\boxed{\overline{Nu_D} = \frac{\bar{h}D}{k_f} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}}$$

↳ Cylinder, $Re_D > 10^4$

$$\boxed{\overline{Nu_D} = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}}$$

⇒ Sphere
$3.5 < Re_D < 8\times10^4$
$0.7 < Pr < 380$

For rest of correlations, see pg. 123 of notes.

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

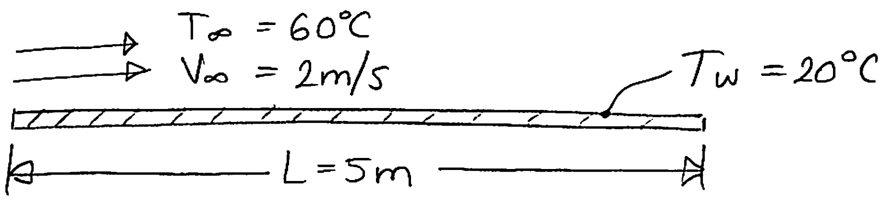
Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668\,Gz_D}{1 + 0.04\,Gz_D^{2/3}}$	(8.57)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s, $Gz_D = (D/x)\,Re_D\,Pr$
$\overline{Nu}_D = \dfrac{\dfrac{3.66}{\tanh[2.264\,Gz_D^{-1/3} + 1.7\,Gz_D^{-2/3}]} + 0.0499\,Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\,Pr^{1/6}\,Gz_D^{-1/6})}$	(8.58)	Laminar, combined entry, $Pr \gtrsim 0.1$, uniform T_s, $Gz_D = (D/x)\,Re_D\,Pr$
$\dfrac{1}{\sqrt{f}} = -2.0 \log\left[\dfrac{e/D}{3.7} + \dfrac{2.51}{Re_D\sqrt{f}}\right]$	(8.20)[c]	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023\,Re_D^{4/5}\,Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
$Nu_D = \dfrac{(f/8)(Re_D - 1000)\,Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D\,Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'', $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D\,Pr \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D\,Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Re_D\,Pr \gtrsim 100$

[a]The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.
[b]Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f = (T_s + T_m)/2$; properties in Equations 8.57 and 8.58 are based on $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[c]Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.
[d]As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[e]For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

NOTE* For a summary of external flow correlations, go to page # 123 of the notes (Table 7.7 or 7.9)

Nenad Miljkovic

Example Engine oil at 60°C flows over a 5m long flat plate (20°C) with velocity of 2m/s. Determine total drag force and heat transfer per unit width.



$$T_f = \bar{T} = \frac{T_W + T_\infty}{2} = 40°C$$

$\rho_{oil} = 876 \ kg/m^3$
$k_{oil} = 0.144 \ W/m \cdot K$
$Pr = 2870$
$\nu_{oil} = 242 \times 10^{-6} \ m^2/s$

Lets check our flow regieme:

$$Re_L = \frac{V_\infty L}{\nu_{oil}} = 4.13 \times 10^4 < 5 \times 10^5 \ (\text{Laminar})$$

We know from pg. 123 of notes:

$$\bar{C_f} = \frac{\bar{\tau_o}}{\frac{1}{2}\rho V_\infty^2} = 1.328 \ Re_L^{-0.5} = 0.00653$$

$$F_D = \bar{C_f} \cdot A \cdot \frac{\rho V_\infty^2}{2} = (0.00653)(5m \cdot 1m)\frac{(876 \ kg/m^3)(2m/s)^2}{2}$$

$$\boxed{F_D = 57.2 \ N}$$

Nenad Miljkovic

For heat transfer:

$$\overline{Nu_L} = \frac{\bar{h} L}{k_{oil}} = 0.664\, Re_L^{1/2}\, Pr^{1/3} = 1918$$

$$\bar{h} = 55.2\ W/m^2 \cdot K$$

$$\boxed{Q = \bar{h} A (T_\infty - T_w) = 11.04\ kW}$$

Now what if we increased the flow rate by 15X..
Do we expect friction & heat transfer to increase by 15x?

$$Re_L = \frac{V_\infty L}{\nu_{oil}} = 6.2 \times 10^5 > Re_{CR} \quad \text{(Turbulent flow)}$$

$$\overline{C_f} = 0.074\, Re_L^{-1/5} - 1742\, Re_L^{-1} \quad (Re_L < 10^8) \Rightarrow \text{Mixed flow}$$

$$\overline{C_f} = 0.00233$$

$$F_D = \overline{C_f} \cdot A \cdot \frac{\rho V_\infty^2}{2} = (0.00233)(5m^2)\frac{(876\ kg/m^3)(30\ m/s)^2}{2}$$

$$\boxed{F_D = 4.59\ kN}$$

For heat transfer:

$$\overline{Nu_L} = \frac{\bar{h} L}{k_{oil}} = (0.037\, Re_L^{4/5} - 871)\, Pr^{1/3} = 10\,255.2$$

$$\bar{h} = 295.3\ W/m^2 \cdot K$$

$$\boxed{Q = \bar{h} A (T_\infty - T_w) = 59.1\ kW}$$

$$\boxed{\frac{F_{D,TURB}}{F_{D,LAM}} = 80.2}$$

$$\boxed{\frac{Q_{TURB}}{Q_{LAM}} = 5.4}$$