Nenad Miljkovic

What if we picked a different working fluid such as water

For <u>water</u>: $\rho_w = 1000\ kg/m^3$
$k_w = 0.6\ W/m\cdot K$
$Pr = 7$
$\nu_w = 0.658\times10^{-6}\ m^2/s$

$$Re_L = \frac{V_\infty L}{\nu_w} = \frac{(2m/s)(5m)}{(0.658\times10^{-6}m^2/s)} = 1.52\times10^{7} \quad (\text{Turbulent!})$$

$$\overline{C_f} = 0.074\,Re_L^{-1/5} - 1742\,Re_L^{-1} = 0.00259$$

$$F_D = \overline{C_f}\cdot A\cdot\frac{\rho_w V_\infty^2}{2} = (0.00259)(5m^2)\frac{(1000kg/m^3)(2m/s)^2}{2}$$

$$\boxed{F_D = 25.9\ N}$$

For heat transfer:

$$\overline{Nu_L} = \frac{\bar{h}L}{k_w} = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3} = 35593.8$$

$$\bar{h} = \frac{\overline{Nu_L}\cdot k_w}{L} = 4271.3\ W/m^2\cdot K$$

$$\boxed{Q = \bar{h}A(T_w - T_\infty) = 0.854\ MW}\ !!!$$

$$\boxed{\frac{F_{D,oil}}{F_{D,w}} = 2.2}\ ; \quad \boxed{\frac{Q_{oil}}{Q_w} = 0.013 \text{ or } 1.3\% \text{ only!}}$$

This simple analysis shows the importance of selecting the appropriate working fluid.

You may ask yourself, why would anybody use high Pr oils for thermal applications? Isn't water way better?

The answer is NO! Oils have 2 main advantages:

1) Temperature range (Typically $-40°C$ to $300°C$).

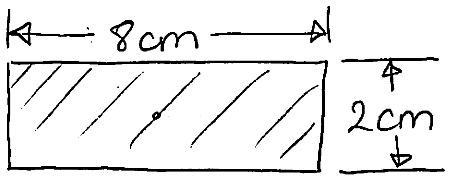
2) Low volatility. Water has a relatively high vapor pressure, meaning it will evaporate easily.

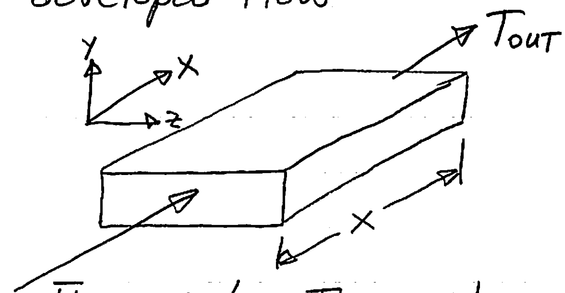
3) Corrosion resistance and enhanced lubrication.

4) Oil is a dielectric fluid (electricaly insulating). Water is also a dielectric, however disolved ions can readily make it into an electrolyte.

Nenad Miljkovic

Example | Heat exchange to a liquid flowing in a channel with T_w = constant and fully developed flow:





$\bar{u} = 0.05 m/s$, T_{in}, water

Determine T_{OUT} as a function of x (length of channel)

First let's find the flow regieme:

$$Re_D = \frac{\rho \bar{u} D}{\mu} \Rightarrow \text{But what } D \text{ do we use?}$$

We need hydraulic diameter:

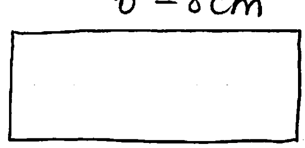
$$D_h = \frac{4A}{P} = \frac{4(8cm)(2cm)}{2(2cm)+2(8cm)} = \frac{64 cm^{\cancel{2}}}{20 \cancel{cm}} = 3.2cm$$

$$Re_{D_h} = \frac{\rho_w D_h \bar{u}}{\mu_w} = \frac{(1000 kg/m^3)(0.032m)(0.05m/s)}{(0.001 Pa \cdot s)} = 1600$$

$$Re_{D_h} = 1600 < 2300 \Rightarrow \text{LAMINAR Flow}$$

So to solve for h, now we must look at table 8.1 on page of the class notes: (since non-circular cross section)

$b = 8cm$



$a = 2cm$

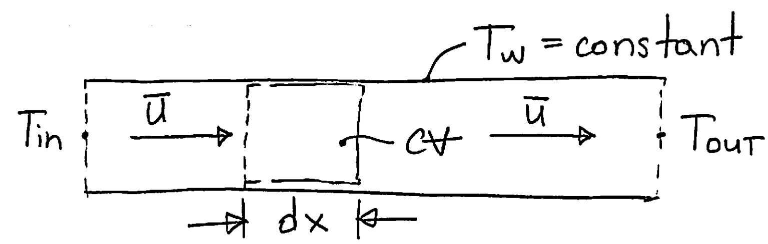
$$\frac{b}{a} = \frac{8}{2} = 4.0, \quad Nu_D = \frac{h D_h}{k_f} = 4.44$$

Uniform T_s

Nenad Miljkovic

$$h = \frac{Nu_D \cdot k_f}{D_h} = \frac{(4.44)(0.6\,W/m\cdot K)}{0.032m} = 83.25\,W/m^2\cdot K$$

So now we can solve our problem. Let's look at a differential axial section of our channel:



Doing an energy ballance on our control volume:

$$\underbrace{\rho A \bar{u} c}_{\dot{m}}\, dT_f = h \underbrace{P\, dx}_{dA_{surface}} (T_w - T_f) \; ; \quad \begin{array}{l} c = \text{heat capacity} \\ A = \text{cross-sectional area} \\ P = \text{perimeter} \end{array}$$

We want to convert this equation into a homogeneous ODE
Let's non-dimensionalize:

Let: $$T^* = \frac{T_f - T_w}{T_{in} - T_w} \; ; \quad x^* = \frac{x}{L} \; ; \quad L = \text{Length of channel}$$

$$dT^* = \frac{dT_f}{T_{in} - T_w} \; ; \quad dx^* = \frac{dx}{L}$$

Back substitute into our ODE:

$$\rho A \bar{u} c \cancel{(T_{in} - T_w)}\, dT^* = hPL\, dx^* (-T^*)\cancel{(T_{in} - T_w)}$$

$$\rho A \bar{u} c\, dT^* = -hPLT^* dx^*$$