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We can solve this homogeneous ODE with integration

$$\int_{T^*_{in}=0}^{T^*_{OUT}} \frac{dT^*}{T^*} = -\frac{hPL}{\rho A c \bar{u}} \int_0^1 dx^*$$

$$\ln\left(\frac{T^*}{T^*_{in}}\right) + \frac{hPLx^*}{\rho A c \bar{u} c} = 0$$

$$T^* = T^*_{in} e^{-\frac{hPLx^*}{\rho A c \bar{u}}} \Rightarrow T^*_{in} = \frac{T_{in} - T_w}{T_{in} - T_w} = 1$$

$$\boxed{T_f = T_w + \left(T_{in} - T_w\right) e^{-\frac{hPx}{\rho A \bar{u} c}}}$$

↳ Bulk fluid temperature as a function of x along the length of the rectangular channel.

Also remember: $T_f = T_b$ = bulk fluid temperature

i.e. $$T_b = \frac{1}{\rho A \bar{u} c} \int_A \rho c u(z,y) \cdot T(z,y) dA$$

or T_f

Note also: $dQ = hP dx \left(T_w - T_b\right)$

↳ $h = f(k_f) = f(T_b)$ ⇒ fluid properties are temperature dependent

For problems like these (internal flow), evaluate properties at $\bar{T}$

$$\boxed{\bar{T} = \frac{T_{in} + T_{OUT}}{2}}$$

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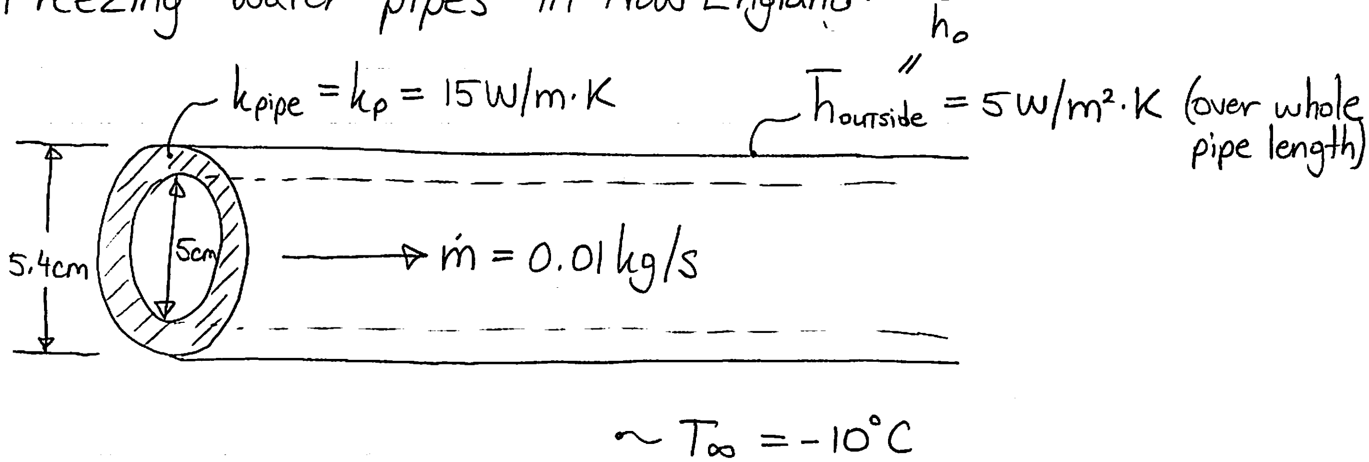
Example | Let's do a comprehensive example that uses everything we've learned so far in class (conduction & convection)

Freezing water pipes in New England:



$T_{in} = 1.5°C$ (inlet pipe fluid bulk temperature)

(a) Calculate the heat transfer coefficient on the inside of the pipe ($\bar{h}_i$). Assume fully developed flow (both thermally and hydrodynamically), and no ice is present.

First let's see our flow regieme:

$$\dot{m} = \frac{\rho_w \pi D_{in}^2 \bar{u}}{4} \Rightarrow \text{from tables, } \rho_w = 1000\,kg/m^3,\quad \nu_w = 1.5\times10^{-6}\,m^2/s,\quad k_w = 0.6\,W/m\cdot K$$

$$Re_D = \frac{\bar{u} D_{in}}{\nu_w} = \frac{4\dot{m}}{\pi \rho_w D_{in} \nu_w} = 170 \text{ (LAMINAR FLOW)}$$

So we know $\overline{Nu_D}$ = constant, however we don't have constant wall temperature or constant heat flux. As a rough approximation, we can use the average:

$$Nu_D \cong \frac{1}{2}(3.657 + 4.364) = 4.01 \Rightarrow \boxed{\bar{h}_i = \frac{\overline{Nu_D}\cdot k_w}{D_{in}} = 45.7\ W/m^2\cdot K}$$

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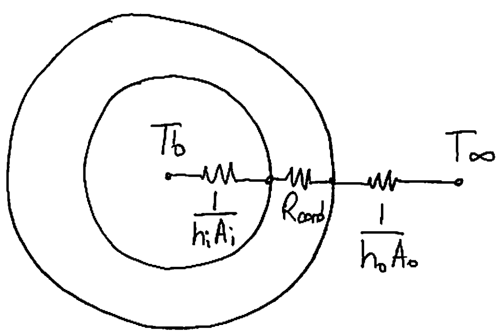
(b) Using our calculated $\bar{h}$, determine the bulk water temp. T_b at the point at which water first freezes.

To solve this question, we need to understand the overall heat transfer from the outside air, to the bulk fluid.



$$\underbrace{\frac{1}{\pi D_{in} U}}_{\text{Overall thermal resistance}} = \frac{1}{\pi D_{in} \bar{h}_i} + \frac{\ln(D_{OUT}/D_{in})}{2\pi k_p} + \frac{1}{\pi D_{OUT} \bar{h}_o}$$

We can solve for the overall heat transfer coefficient U

$$U = 4.20 \text{ W/m}^2\cdot\text{K}$$

Now we can solve for when water begins to freeze:

$$q = \bar{h}_i (T_b - T_{freeze}) = U (T_b - T_\infty)$$

$$T_{freeze} = 0°C$$

Rearanging: $$\boxed{T_b = \frac{\bar{h}_i T_{freez} - U T_\infty}{\bar{h}_i - U} = 1.20°C}$$

(c) At what point in the pipe length L does the water begin to freeze?

Applying an energy balance on a differential fluid element just like the previous example:

$$\dot{m}c\frac{dT_b}{dx} = \pi D_{in} U (T_\infty - T_b)$$

We already solved that:

$$\frac{T_b - T_\infty}{T_{in} - T_\infty} = \exp\left(-\frac{\pi D_{in} U L}{\dot{m}c}\right) \quad ; \text{ note } T_{in} = \text{inlet fluid temperature}$$

From this we can solve for L given that $T_b = 1.20°C$, $T_\infty = -10°C$, $T_{in} = 1.5°C$, $C = 4200\ J/kg \cdot K$ (from Tables)

$$\boxed{L = \frac{\dot{m}c}{\pi D_{in} U} \cdot \ln\left(\frac{T_{in} - T_\infty}{T_b - T_\infty}\right) = 1.47\,m}$$

(d) Suppose the flow rate is increased to $\dot{m} = 1\ kg/s$. At a point $L = 10m$ from the entrance, determine the temperature on the inner wall of the pipe. Assume same conditions as before.

First let's check our flow regieme again:

$$Re_{D_n} = \frac{4\dot{m}}{\pi \rho_w D_{in} \nu_w} = 1.7\times10^4 > 2300 \text{ (Turbulent)}$$

So we need to use the Gnielinski correlation:

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$$f = (0.79 \ln Re_D - 1.64)^{-2} = 0.0273$$

$$\overline{Nu_D} = \frac{h_i D_{in}}{k_f} = \frac{(f/8)(Re_D - 1000) Pr_w}{1 + 12.7 (f/8)^{1/2} (Pr_w^{2/3} - 1)}$$

From Tables, $Pr_w \cong 11$

$$\overline{Nu_D} = 153 = \frac{\bar{h}_i D_{in}}{k_f} \Rightarrow \bar{h}_i = 1610\ \text{W/m}^2\cdot\text{K}$$

We can now re-calculate our overall heat transfer coefficient, U:

$$U = \left\{ \pi D_{in} \left[\frac{1}{\pi D_{in} \bar{h}_i} + \frac{\ln(D_{out}/D_{in})}{2\pi k_p} + \frac{1}{\pi D_{out} \bar{h}_o} \right] \right\}^{-1}$$

$$U = 5.38\ \text{W/m}^2\cdot\text{K}$$

Now we use our energy balance equation to solve for T_b at $L = 10\text{m}$

$$T_b = T_\infty + (T_{in} - T_\infty)\exp\left[-\frac{\pi D_{in} U L}{\dot{m} c}\right] = 1.48^\circ\text{C}$$

Finally, using thermal resistance to solve for inner surface temperature: $(T_{s,i})$

$$q = \bar{h}_i (T_b - T_{s,i}) = U(T_b - T_\infty)$$

$$\boxed{T_{s,i} = \frac{(\bar{h}_i - U) T_b - U T_\infty}{\bar{h}_i} = 1.44^\circ\text{C}}$$

So we see that one mitigation strategy to prevent icing is to increase the flow rate.