

Total Emissive Power

If we want total power emitted by a radiating body over all wavelengths, we need to integrate over the whole spectrum:

$$e_b = \int_0^{\infty} e_{b,\lambda} d\lambda = T^5 C_1 \int_0^{\infty} \frac{(\lambda T)^{-5}}{\exp(C_2/\lambda T) - 1} d\lambda$$

$$= T^4 \left[C_1 \int_0^{\infty} \frac{\eta^5}{e^{C_2/\eta} - 1} d\eta \right]; \quad \eta = \lambda T$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

↳ Stefan-Boltzmann constant


So our total emissive power is simply:

$$e_b = \sigma T^4 \Rightarrow T \text{ is in Kelvin!}$$

↳ This is a heat flux [W/m²].

Note, the previous analysis is only valid for ideal emitters.

Ideal Emitter \Leftrightarrow "Blackbody"

- 1) Blackbodies absorb all incoming radiation
- 2) Radiation is independent of direction (Diffuse) 
- 3) At a given temperature, T , no surface can emit more energy than a blackbody.

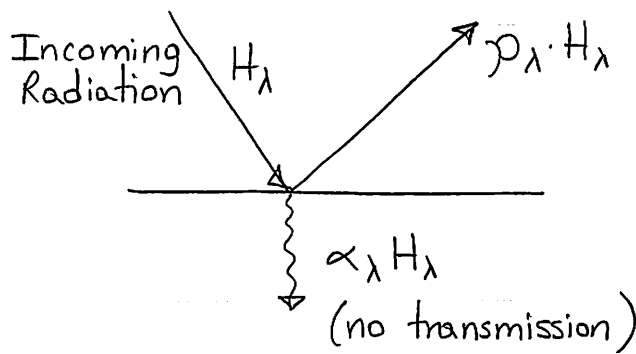
For non-blackbody surfaces (real surfaces), we can define a monochromatic emissivity, ϵ_λ

$$e_\lambda = \epsilon_\lambda \cdot e_{b,\lambda}$$

$$e = \int_0^\infty \epsilon_\lambda e_{b,\lambda} d\lambda = \epsilon \cdot e_b$$

$$\boxed{\epsilon = \frac{\int_0^\infty \epsilon_\lambda \cdot e_{b,\lambda} \cdot d\lambda}{\int_0^\infty e_{b,\lambda} d\lambda}} \Rightarrow \text{Spectrally averaged emissivity}$$

In real life, we also have reflection (ρ) and absorption (α)



An energy balance:

$$\boxed{\rho_\lambda + \alpha_\lambda = 1}$$

In thermal equilibrium:

$$\boxed{\epsilon_\lambda = \alpha_\lambda} \text{ Kirchoff's Law}$$

Similarly for absorption:

$$\boxed{\alpha = \frac{\int_0^\infty \alpha_\lambda H_\lambda d\lambda}{\int_0^\infty H_\lambda d\lambda}} \Rightarrow \text{Spectrally averaged absorptivity}$$

From Kirchoff's Law we can say:

- 1) Good absorbers are good emitters
 - 2) Poor absorbers are poor emitters
- } Experimentally observed.

Note, if we have transmission (τ_λ) $\Rightarrow \boxed{\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1}$

Example | A car accident injures a passenger at night and to help save body heat, the EMT's drape the passenger in an aluminum blanket. Assume $T_{\infty} = -50^{\circ}\text{C}$, and $\epsilon_{\text{blanket}} = 0.10$. How much heat loss is saved?

This is a very common technique to limit radiative losses, especially at night:

Assume your body temperature is at 37°C

$$e_b = \sigma T^4 = (5.67 \times 10^{-8}) \left[(37 + 273.15)^4 - (-50 + 273)^4 \right]$$

$e_b = 384.05 \text{ W/m}^2$

\uparrow
 emission from
 atmosphere to you.

Now if we drape the aluminum blanket:

$$e_b = \epsilon_{\text{blanket}} \cdot \sigma T^4 = (0.10) (5.67 \times 10^{-8}) (310^4 - 223^4)$$

$e_b = 38.3 \text{ W/m}^2$

So how does this compare to convection? Let's assume $T_{\text{air}} \approx 5^{\circ}\text{C}$, and $h_{\text{conv}} = 20 \text{ W/m}^2 \cdot \text{K}$ (light breeze)

$$q''_{\text{conv}} = h \Delta T = (20 \text{ W/m}^2 \cdot \text{K}) (37^{\circ}\text{C} - 5^{\circ}\text{C})$$

$q''_{\text{conv}} = 640 \text{ W/m}^2$

So we can see that both radiation & convection are very important mechanisms and that draping the reflective aluminum blanket lowers losses by about 35%.