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View Factors and Blackbody Heat Exchange

So far, everything has been about blackbody radiation to an infinite medium. I.e.: (surfaces see eachother)

$e_{b,\infty} = \varepsilon\sigma T_\infty^4$

$\varepsilon = 1$ (blackbody)

$e_b = \varepsilon\sigma T_B^4$

$$\boxed{e_{b,net} = \sigma\left(T_B^4 - T_\infty^4\right)}$$

$$\neq \sigma\left(T_b - T_\infty\right)^4 \; !$$

But, what if we have finite surfaces:

$$\left.\begin{aligned} q_{1\to2} &= F_{12} A_1 e_{b1} \\ q_{2\to1} &= F_{21} A_2 e_{b2} \end{aligned}\right\}$$

F_{ab} ≡ view factor
≡ fraction of radiation leaving surface "a" and reaching "b".

So our net exchange is:

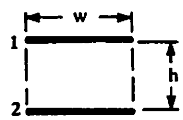
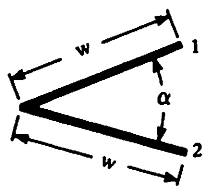
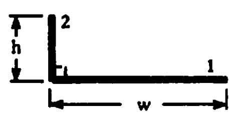
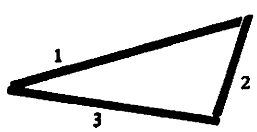
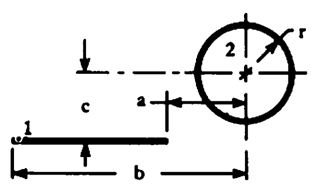
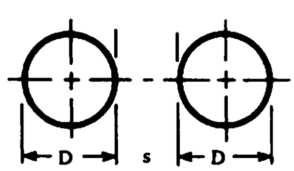
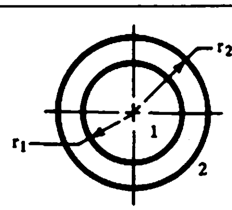
$$q_{12} = q_{1\to2} - q_{2\to1} = F_{12} A_1 e_{b1} - F_{21} A_2 e_{b2}$$

But if $T_1 = T_2$, $e_{b1} = e_{b2}$

$$q_{12} = e_b\left(A_1 F_{12} - A_2 F_{21}\right) = 0$$ (no net energy transfer in between surfaces at same T)

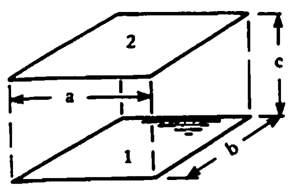
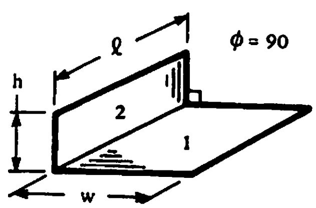
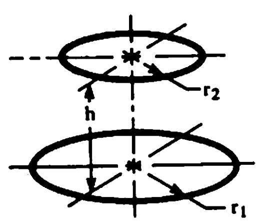
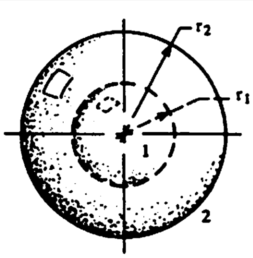
$\boxed{A_1 F_{12} = A_2 F_{21}}$ ⇒ Reciprocity

Table 10.2 View factors for a variety of two-dimensional configurations (infinite in extent normal to the paper)

Configuration		*Equation*
1.		$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$
2.		$F_{1\text{-}2} = F_{2\text{-}1} = 1 - \sin(\alpha/2)$
3.		$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$
4.		$F_{1\text{-}2} = (A_1 + A_2 - A_3)/2A_1$
5.		$F_{1\text{-}2} = \frac{r}{b-a}\left[\tan^{-1}\frac{b}{c} - \tan^{-1}\frac{a}{c}\right]$
6.		Let $X = 1 + s/D$. Then: $F_{1\text{-}2} = F_{2\text{-}1} = \frac{1}{\pi}\left[\sqrt{X^2 - 1} + \sin^{-1}\frac{1}{X} - X\right]$
7.		$F_{1\text{-}2} = 1,\quad F_{2\text{-}1} = \frac{r_1}{r_2}$, and $F_{2\text{-}2} = 1 - F_{2\text{-}1} = 1 - \frac{r_1}{r_2}$

Adapted From Lienhard & Lienhard "A Heat Transfer Textbook" (2012)

Table 10.3 View factors for some three-dimensional configurations

Configuration	*Equation*
1. 	Let $X = a/c$ and $Y = b/c$. Then: $$F_{1\text{-}2} = \frac{2}{\pi XY}\left\{\ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} - X\tan^{-1}X - Y\tan^{-1}Y + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}}\right\}$$
2. 	Let $H = h/\ell$ and $W = w/\ell$. Then: $$F_{1\text{-}2} = \frac{1}{\pi W}\left\{W\tan^{-1}\frac{1}{W} - \sqrt{H^2+W^2}\tan^{-1}\left(H^2+W^2\right)^{-1/2} + H\tan^{-1}\frac{1}{H} + \frac{1}{4}\ln\left\{\left[\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\right] \times \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2}\left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right\}$$
3. 	Let $R_1 = r_1/h$, $R_2 = r_2/h$, and $X = 1 + \left(1 + R_2^2\right)/R_1^2$. Then: $$F_{1\text{-}2} = \frac{1}{2}\left[X - \sqrt{X^2 - 4(R_2/R_1)^2}\right]$$
4. 	Concentric spheres: $F_{1\text{-}2} = 1$, $F_{2\text{-}1} = (r_1/r_2)^2$, $F_{2\text{-}2} = 1 - (r_1/r_2)^2$

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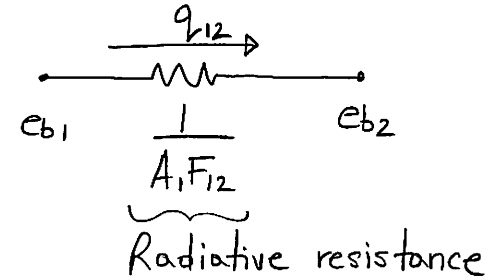
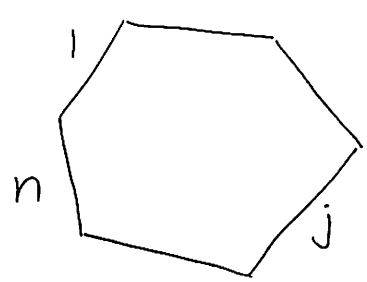
Now we can formulate our analysis as:

$$q_{12} = \underbrace{A_1 F_{12} \left(e_{b1} - e_{b2}\right)}_{A_1 F_{12} e_{b1} - A_2 F_{21} e_{b2}} \iff$$



For <u>enclosures</u>:



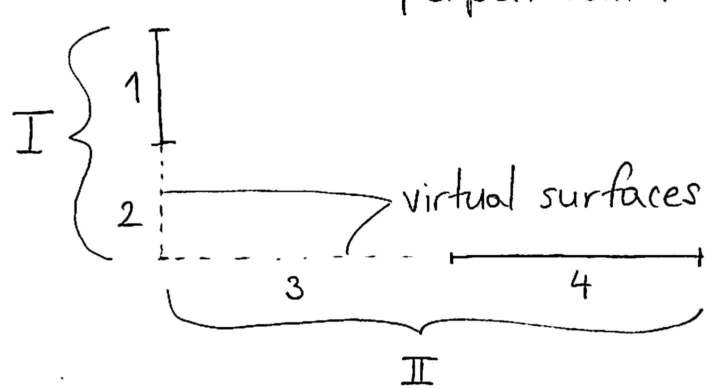
$$\boxed{\sum_{j=2}^{n} F_{1j} = 1} \Rightarrow \text{True for flat or convex surfaces.}$$

If concave surface

$$\sum_{j=2}^{n} F_{1j} \neq 1 \quad \text{since } F_{11} \neq 0$$

(Surface sees itself)

Example Consider the following arrangement between two perpendicular plates 1 & 4



Can look these up in view factor tables.

$$A_1 F_{14} = A_I F_{I4} - A_2 F_{24}$$

$$A_I F_{I4} = A_I F_{I\,II} - A_I F_{I3} \Rightarrow F_{I4} = F_{I\,II} - F_{I3}$$

$$A_2 F_{24} = A_2 F_{2\,II} - A_2 F_{23} \Rightarrow F_{24} = F_{2\,II} - F_{23}$$