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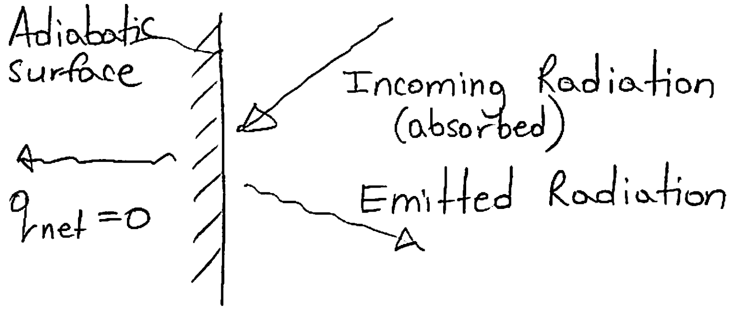
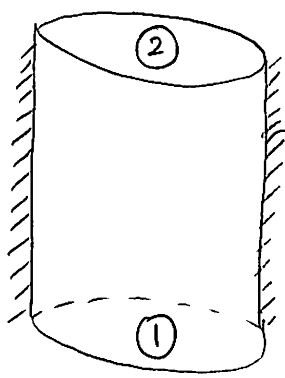
Adiabatic Surfaces (no heat transfer through that surface)



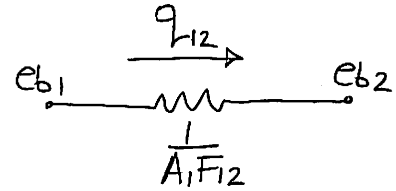
} Must be equal for $q_{net} = 0$

If we look at an enclosure: (furnace)

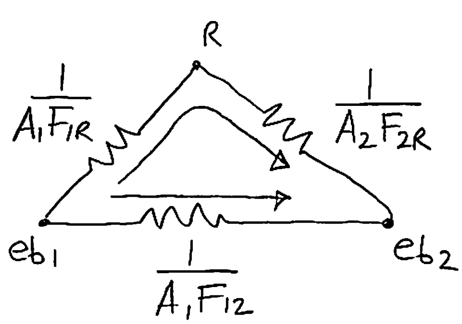


Adiabatic Wall; R } $q'' = 0$

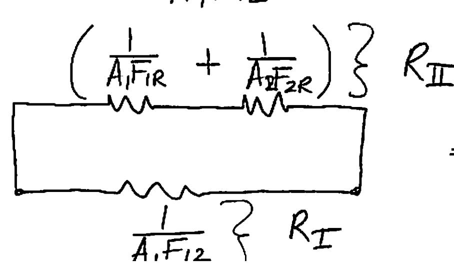
Previously we had:



Now we have:



We can redraw this:



$$\Rightarrow R_{TOT} = \left[\frac{1}{R_I} + \frac{1}{R_{II}}\right]^{-1}$$

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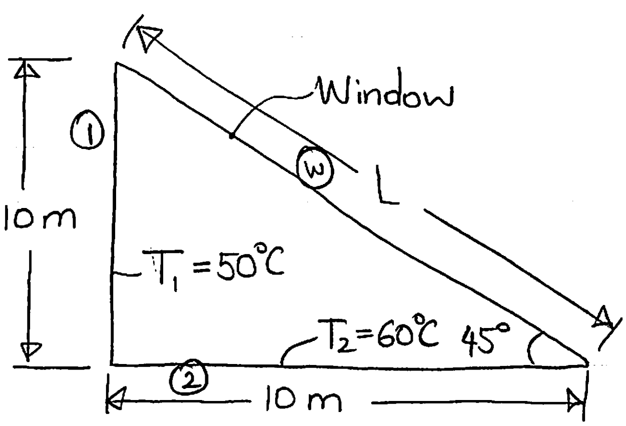
$$\frac{1}{R_{TOT}} = \frac{1}{R_{I}} + \frac{1}{R_{II}}$$

$$= A_1 F_{12} + \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}}}$$

$$q_{12} = \frac{e_{b1} - e_{b2}}{R_{TOT}}$$

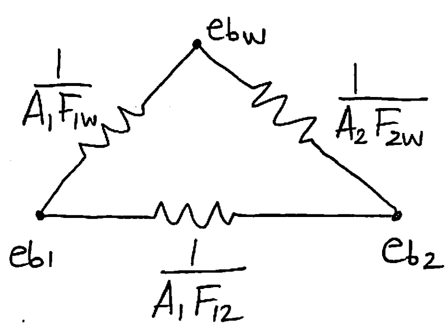
$$\boxed{q_{12} = (e_{b1} - e_{b2})\left[A_1 F_{12} + \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}}}\right]}$$ ⇒ Units of Watts

Example | A solar greenhouse with a triangular architecture has the following design:



Assuming the widow is a perfect re-radiator, find the window temperature.

First, we can draw our radiative resistance network.



From symmetry:
$A_1 = A_2$
$F_{12} = F_{21}$
$F_{1W} = F_{2W}$

$$q_{12} = \frac{e_{b1} - e_{b2}}{R_{TOT}} = (e_{b1} - e_{b2})\left[A_1 F_{12} + \frac{1}{\frac{1}{A_1 F_{1W}} + \frac{1}{A_2 F_{2W}}}\right]$$

Looking back at our view factor tables: (pg. 163 of notes, case 2)

$$F_{12} = F_{21} = 1 - \sin\left(\frac{\alpha}{2}\right) \quad ; \quad \alpha = 90^\circ$$

$$F_{12} = F_{21} = 0.292$$

$$F_{1W} = 1 - F_{12} = 0.707$$ (since surface 1 is flat, i.e. $F_{11} = 0$)

$$e_{b1} = \sigma T_1^4 = (5.67\times10^{-8})(50+273.15)^4 = 618.3\ W/m^2$$

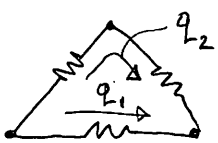
$$e_{b2} = \sigma T_2^4 = (5.67\times10^{-8})(60+273.15)^4 = 698.46\ W/m^2$$

$$q_{12} = (e_{b1} - e_{b2})\underbrace{\left[(10m)(0.292) + \frac{1}{\frac{1}{(10m)(0.707)} + \frac{1}{(10m)(0.707)}}\right]}_{6.455}$$

$$= (e_{b1} - e_{b2})(6.455)$$

$\boxed{q_{12} = -516.7\ W/m}$ ⇒ Heat radiates from ② to ① actually

Now we can solve for T_w. We first need to solve for heat transfer through each leg:



$$q_1 = (e_{b1} - e_{b2})\left(\frac{1}{A_1 F_{12}}\right)^{-1} = -234.1\ W/m$$

$$q_2 = q_{12} - q_1 = -516.7 - (-234.1) = -282.6\ W/m$$

$$(e_{b1} - e_{bw})\left(\frac{1}{A_1 F_{1W}}\right)^{-1} = q_2 \Rightarrow \boxed{T_w = 55^\circ C}$$

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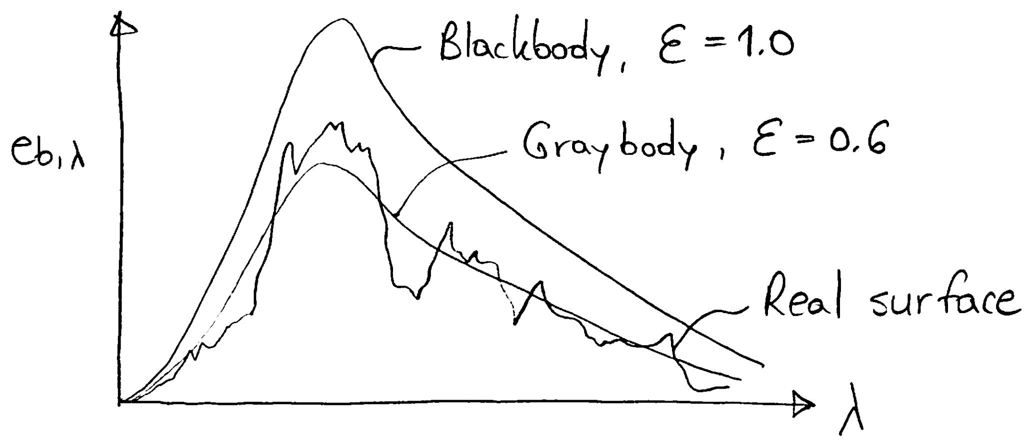
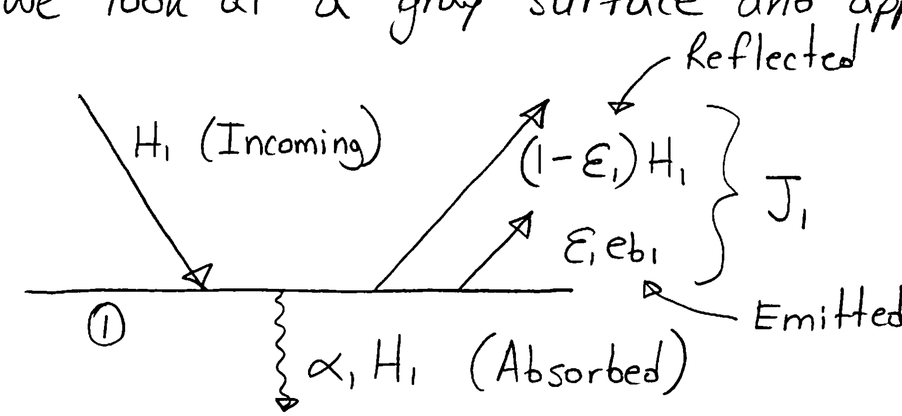
Gray Body Radiation

So far, we've only been dealing with cases where $\varepsilon = 1$ or blackbodies. Most real surfaces fail to satisfy this.

For example:



If we look at a gray surface and apply an energy bal:



Here, we assume Kirchoff's law: $\varepsilon = \alpha$

$H_1 \equiv$ irradiance $[W/m^2]$ (Incoming radiation)

$J_1 \equiv$ radiosity $[W/m^2]$ (Total radiant flux away from the surface, emitted + reflected

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Let's look at each component:

Incoming $\equiv H_1$

Absorbed $\equiv \alpha_1 H_1 = \varepsilon_1 H_1$

Reflected $\equiv$ Incoming $-$ Absorbed $= H_1 - \alpha_1 H_1 = H_1(1-\varepsilon_1)$

Emitted $\equiv \varepsilon_1 e_{b1} = \varepsilon_1 \sigma T_1^4$

So our radiosity (J_1) becomes:

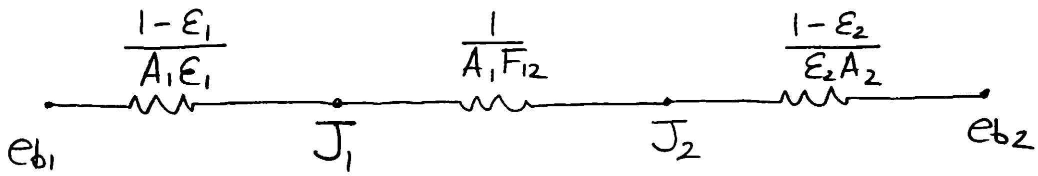
$$J_1 = (1-\varepsilon_1)H_1 + \varepsilon_1 e_{b1}$$ $\Rightarrow$ Rearange this

$$H_1 = \frac{J_1 - \varepsilon_1 e_{b1}}{1-\varepsilon_1}$$

$$\underbrace{\frac{q_{1,NET}}{A_1}}_{\text{Net radiative energy leaving surface 1 (heat flux)}} = (J_1 - H_1) = J_1 - \frac{J_1 - \varepsilon_1 e_{b1}}{1-\varepsilon_1}$$

$$\frac{q_{1,NET}}{A_1} = \frac{J_1(1-\varepsilon_1) - J_1 + \varepsilon_1 e_{b1}}{1-\varepsilon_1} = \frac{\varepsilon_1}{1-\varepsilon_1}(e_{b1} - J_1)$$

$$\boxed{\frac{q_{1,NET}}{A_1} = \frac{\varepsilon_1}{1-\varepsilon_1}(e_{b1} - J_1)}$$ $\Rightarrow$ NET radiative heat flux leaving surface 1.



Note, if you set $\varepsilon_1 = \varepsilon_2 = 1$, you get the blackbody sol.