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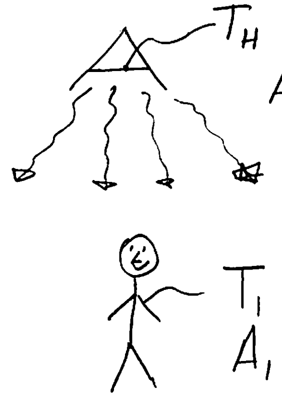
Example | Radiative heater. Find the heat flux to your body assuming:



$T_H = 600°C$
$A_H = 1m^2$

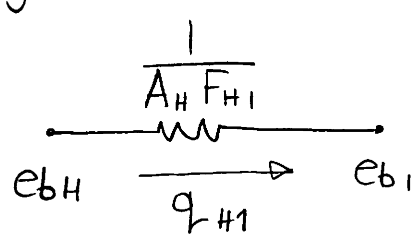
$F_{H1} = 0.01$

$T_1 = 37°C$
$A_1 = 1m^2$

(a) $\varepsilon_H = 1.0$, $\varepsilon_1 = 1.0$ (blackbody)

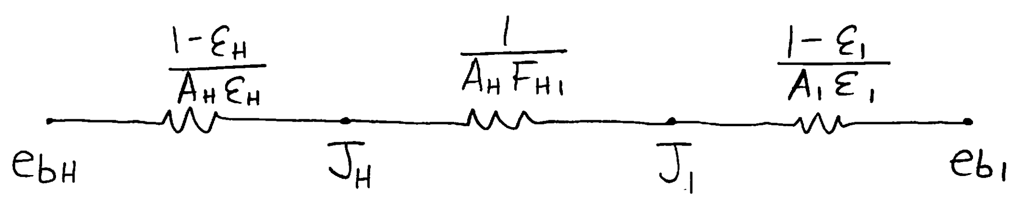
(b) $\varepsilon_H = 0.8$, $\varepsilon_1 = 0.5$ (graybody)

(a) We can right away draw our thermal resistance diagram:



$$\Rightarrow q_{H1} = \frac{e_{bH} - e_{b1}}{\frac{1}{A_H F_{H1}}} = 320\,W$$

(b) Similar approach but more realistic:



$$q_{H1} = \frac{\overbrace{e_{bH} - e_{b1}}^{32431.65}}{\underbrace{\frac{1-\varepsilon_H}{A_H \varepsilon_H}}_{0.25} + \underbrace{\frac{1}{A_H F_{H1}}}_{100} + \underbrace{\frac{1-\varepsilon_1}{A_1 \varepsilon_1}}_{1}} = 319\,W$$

This shows you the importance of relative contribution to resistances. In this case, view factor resistance dominates.

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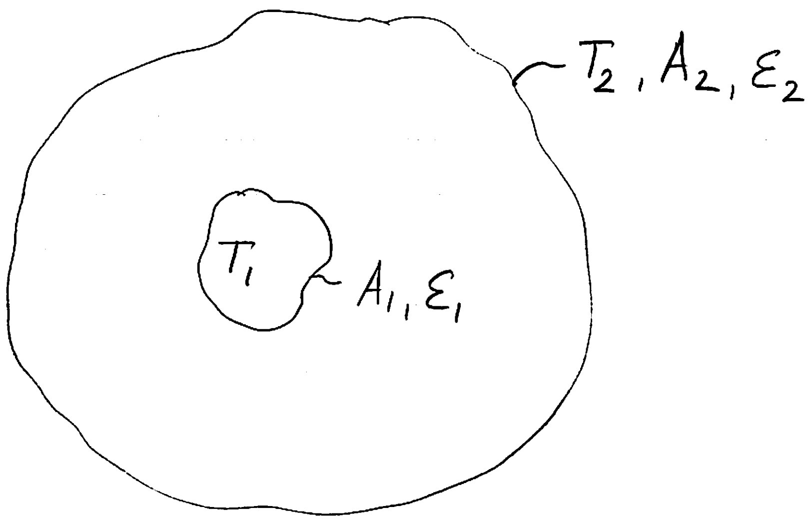
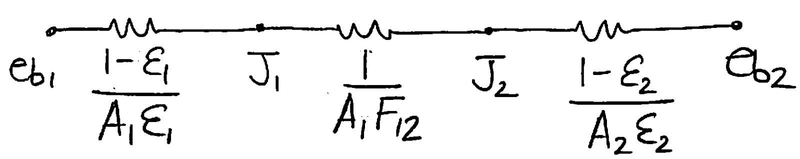
Gray Body Enclosures



Assuming that body 2 is much larger than body 1:



$$q_{1\to 2} = \frac{e_{b1} - e_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} \Rightarrow \text{multiply by } \frac{A_1}{A_1}$$

$$\boxed{q_{1\to 2} = \frac{A_1(e_{b1} - e_{b2})}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

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Some special cases:

(1.) $A_2 \gg A_1$

This means $\frac{A_1}{A_2} \to 0$ and $F_{12} = 1$

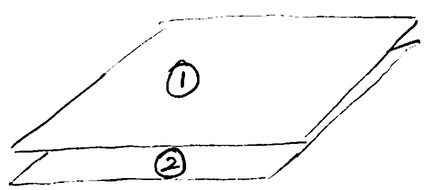
Back substituting into our solution

$$q_{1\to2} = \frac{A_1(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} \cancel{-1} \cancel{+1} + \cancel{0}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$\boxed{q_{1\to2} = A_1 \varepsilon_1 (e_{b1} - e_{b2})}$$

$\Rightarrow$ Smaller body behaves like a black body with finite emissivity.

(2.) Parallel Plates (small gap between them)

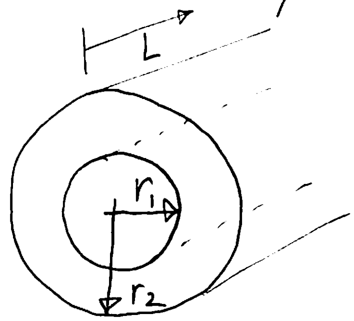


$F_{12} = 1 = F_{21}$
$A_1 = A_2$

$$q_{1\to2} = \frac{A_1(e_{b1} - e_{b2})}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} = \frac{A_1(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} \cancel{-1} \cancel{+1} + \frac{1}{\varepsilon_2} - 1}$$

$$\boxed{q_{1\to2} = \frac{A_1(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

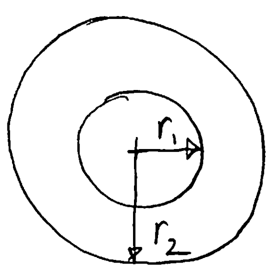
3.) Concentric Cylinders (Length L)



$F_{12} = 1$; $\left. \begin{aligned} A_1 &= 2\pi r_1 L \\ A_2 &= 2\pi r_2 L \end{aligned} \right\}$ $\dfrac{A_1}{A_2} = \dfrac{r_1}{r_2}$

$$\boxed{q_{1\to 2} = \frac{2\pi r_1 L\,(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{r_1}{r_2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

4.) Concentric Spheres

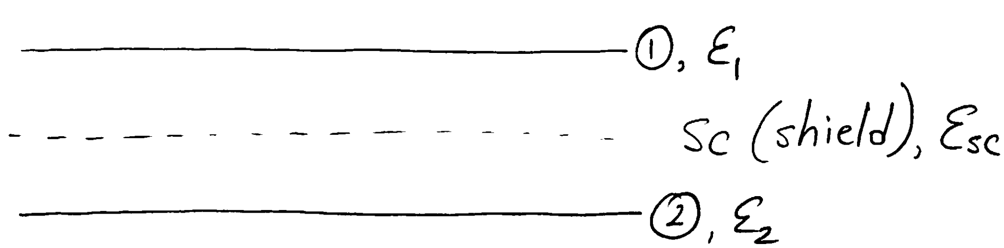


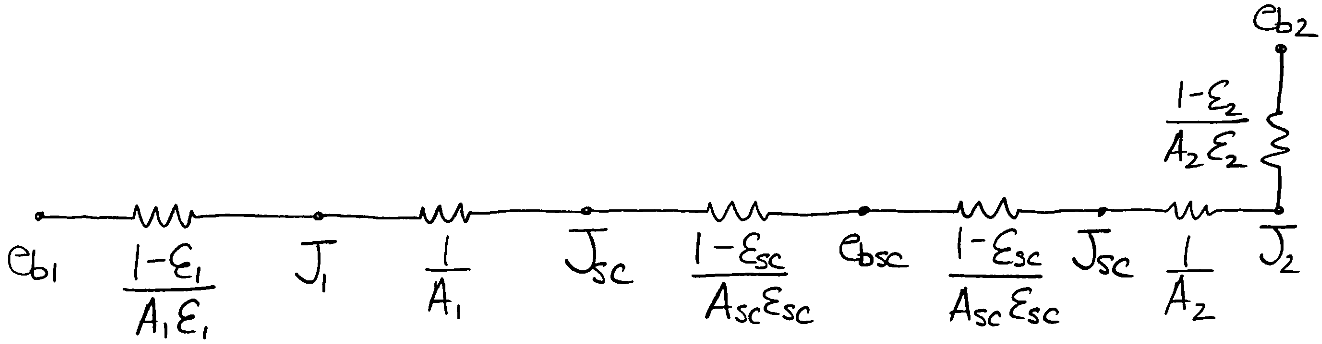
$\dfrac{A_1}{A_2} = \dfrac{r_1^2}{r_2^2}$; $F_{12} = 1$

$$\boxed{q_{1\to 2} = \frac{4\pi r_1^2\,(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{r_1^2}{r_2^2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

Radiation Shields

Used to reduce radiative heat loss
For a flat plate arrangement:





Since $A_1 = A_{sc} = A_2$ (parallel plates)
$F_{12} = F_{sc,2} = F_{2,sc} = 1$

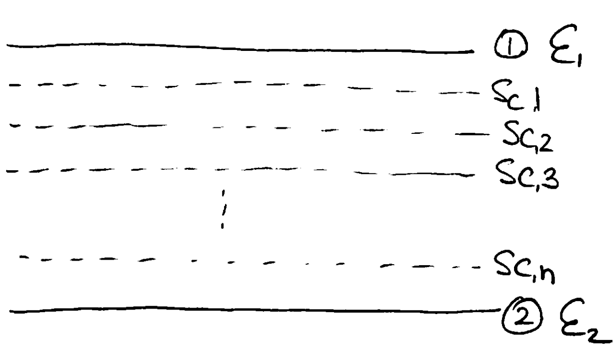
Our result simplifies to:

$$q_{12} = \frac{A\left(e_{b1} - e_{b2}\right)}{\left(\frac{1}{\varepsilon_1} - 1\right) + 1 + \left(\frac{1}{\varepsilon_2} - 1\right) + 2\left(\frac{1}{\varepsilon_{sc}} - 1\right) + 1}$$

$$\boxed{q_{12} = \frac{A\left(e_{b1} - e_{b2}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{2}{\varepsilon_{sc}} - 1\right)}}$$ $\Rightarrow$ For 1 shield

Note the solution is identical to the parallel plate solution from before except now we have an additional term in the denominator that diminishes q_{12}.

For n shields in series: (with identical radiative properties)



$$\boxed{q_{12} = \frac{A\left(e_{b1} - e_{b2}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_{sc}} - (n+1)}}$$

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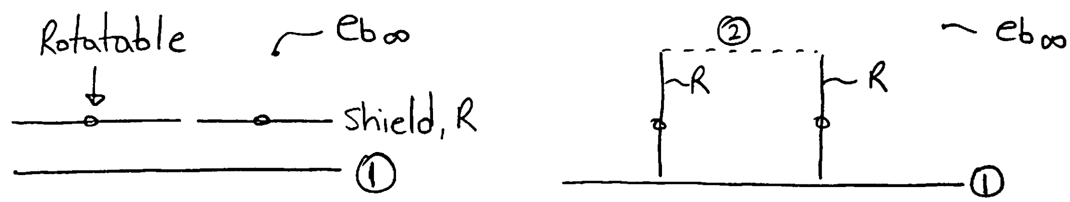
Example | Consider NASA's rotatable radiation shield shown below. Calculate its effectiveness in the on & off positions: $\varepsilon_{sc} = 0.1$, $\varepsilon_1 = 0.5$

State I: ON

State II: OFF



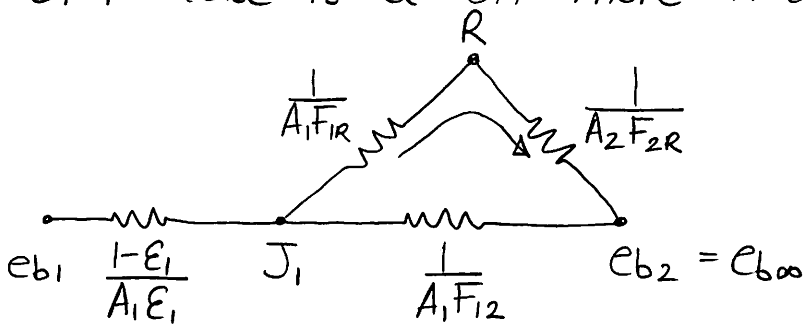
Let's solve the ON case first:

$$e_{b1} \xrightarrow{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} J_1 \xrightarrow{\frac{1}{A_1F_{1sc}}} J_{sc} \xrightarrow{\frac{1-\varepsilon_{sc}}{A_{sc}\varepsilon_{sc}}} e_{bsc} \xrightarrow{\frac{1-\varepsilon_{sc}}{A_{sc}\varepsilon_{sc}}} J_{sc} \xrightarrow{\frac{1}{A_{sc}F_{sc,\infty}}} e_{b\infty}$$

We know $A_1 = A_{sc} = A$ (parallel infinite plates)
$F_{1sc} = F_{sc2} = 1$

$$\boxed{q_{1\infty} = \frac{A_1(e_{b1} - e_{b\infty})}{\frac{1}{\varepsilon_1} + \frac{2}{\varepsilon_{sc}} - 1}} \Rightarrow \text{State I, ON}$$

The OFF case is a bit more tricky to solve:



* The shields in the OFF state act as adiabatic surfaces due to symmetry, so $e_{bR} = J_R$

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From our view factor tables:

$$F_{12} = \sqrt{2} - 1$$

$$F_{1R} = 2 - \sqrt{2}$$

$$\boxed{q_{12} = \frac{A_1(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}}} \Rightarrow \text{State II, OFF}$$

Now we can calculate the ratio of heat transfer from the thermal resistance ratios (denominator):

$$\boxed{\frac{R_I}{R_{II}} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{sc}} - 1}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}} = 13.2}$$

So when the shield is ON, it is 13.2 times better at limiting radiative heat transfer.

Note here the importance of choosing ε_{sc}. If $\varepsilon_{sc} \ll 1$, then radiation shield is much better performing. This is why scientists use highly reflective surfaces for radiation shielding like aluminum foil.