Nenad Miljkovic

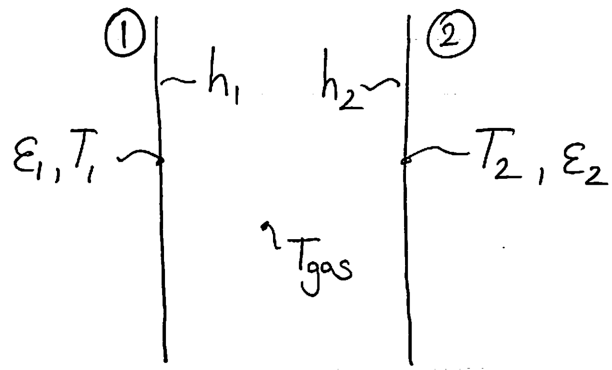
Multi-Mode Heat Transfer

What if we have radiation & convection. How do we handle this?

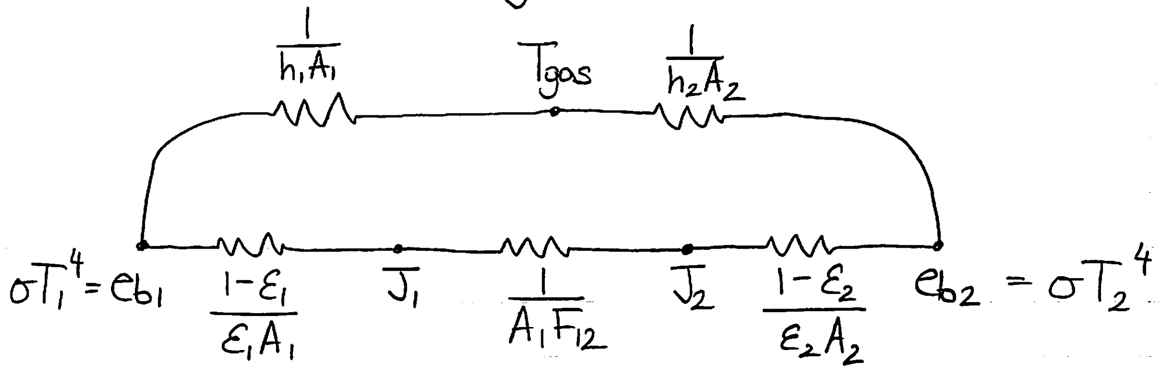
Well, since:

Radiation ⇒ independent of medium
Convection ⇒ medium dependent
} Parallel paths.

For example:

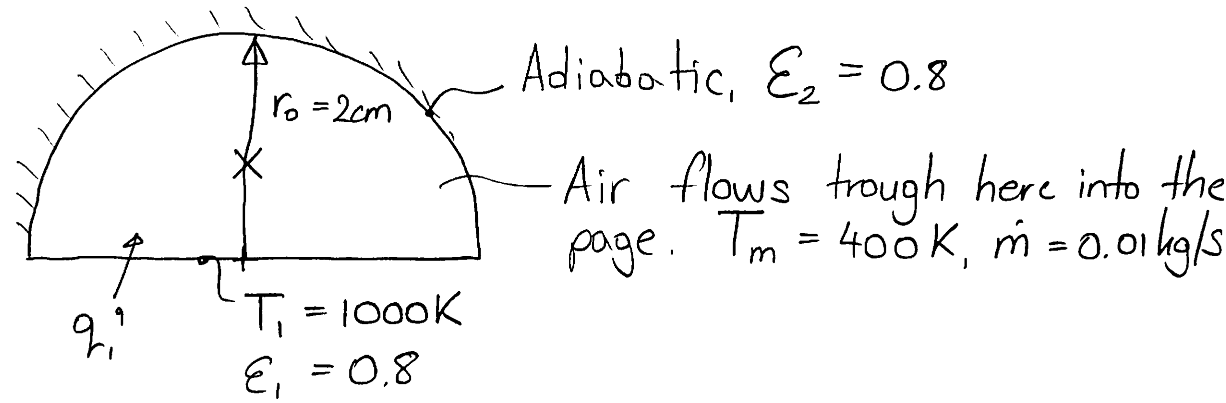


Our resistance diagram becomes:



We need to solve this complete resistance diagram and it usually involves itteration since T_{gas} is unknown.

Example | Consider an air heater: (semi-circular)

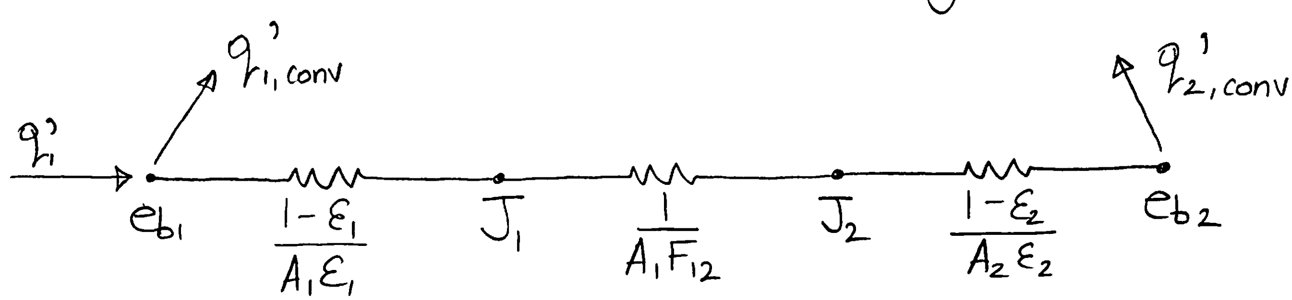


Find the temperature of the adiabatic surface, and the heat required per unit length (q_1') to maintain $T_1 = 1000K$. Assuming fully developed internal flow:

Look up properties in Table A.4 or Incropera (Textbook)

$K_{Air} = 0.0338\ W/m \cdot K$
$\mu = 230 \times 10^{-7}\ kg/(s \cdot m)$
$C_p = 1014\ J/kg \cdot K$
$Pr = 0.69$

Let's draw our thermal resistance diagram:



Since surface 2 is insulated: $q_{2,rad}' = q_{2,conv}'$ (1)

Net radiative transfer into 2.

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We can evaluate the radiative and convective balance on surface 2 as:

Air temperature

$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = h A_2 (T_2 - T_m)$$

We know: $F_{12} = 1$ (doesn't see itself, $F_{11} = 0$)

$A_1 = 2r_o$ (area per unit length)

$A_2 = \pi r_o$ (area per unit length)

Now for the fluids problem: (convection)

$$Re_D = \frac{\rho u D_h}{\mu} = \frac{\dot{m} D_h}{\underbrace{A_c}_{\text{Cross sectional area}} \mu} = \frac{\dot{m} D_h}{(\pi r_o^2/2)\mu}$$

$$D_h = \frac{4A_c}{P} = \frac{2\pi r_o^{\cancel{2}}}{\cancel{r_o}(\pi+2)} = \frac{0.04\pi}{\pi+2} = 0.0244m$$

$$Re_D = \frac{(0.01 kg/s)(0.0244m)}{(\pi/2)(0.02m)^2(230\times10^{-7} kg/m\cdot s)} = 16\,900$$

So we have turbulent fully developed flow.

Flipping back to our correlation table for internal flow: Table 8.4, pg 137 of notes.
We can use (8.60)^d $\Rightarrow$ Dittus-Boelter equation

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$$Nu_D = \frac{h D_h}{k_{air}} = 0.023\, Re_D^{4/5} Pr^{0.4}$$

$$Nu_D = 0.023\,(16900)^{4/5}(0.69)^{0.4} = 47.8$$

$$h = \frac{k_{air} Nu_D}{D_h} = \frac{(0.0338\,W/m\cdot K)(47.8)}{(0.0244m)} = 66.2\,W/m^2\cdot K$$

Now we can work with our energy balance on surface 2.

$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = hA_2(T_2 - T_m) \Rightarrow \text{Divide through by } A_1 = 2r_o,\ A_2 = \pi r_o$$

$$\frac{(5.67\times10^{-8})(1000^4 - T_2^4)}{\frac{1-0.8}{0.8} + 1 + \frac{1-0.8}{0.8}\left(\frac{2}{\pi}\right)} = (66.2)\left(\frac{\pi}{2}\right)(T_2 - 400)$$

Simplifying, we obtain:

$$(5.67\times10^{-8})T_2^4 + 146.5T_2 - 115313 = 0$$

Using itteration (guess T_2 & check solution), we obtain:

$$\boxed{T_2 = 696\,K}$$

Now we can solve for q_1' at the heated surface 1.

An energy balance reveals:

$$q_1' = q_{1,rad}' + q_{1,conv}' = \underbrace{q_{2,conv}' + q_{1,conv}'}_{\text{All heat in } (q_1') \text{ is taken away by convection.}}$$

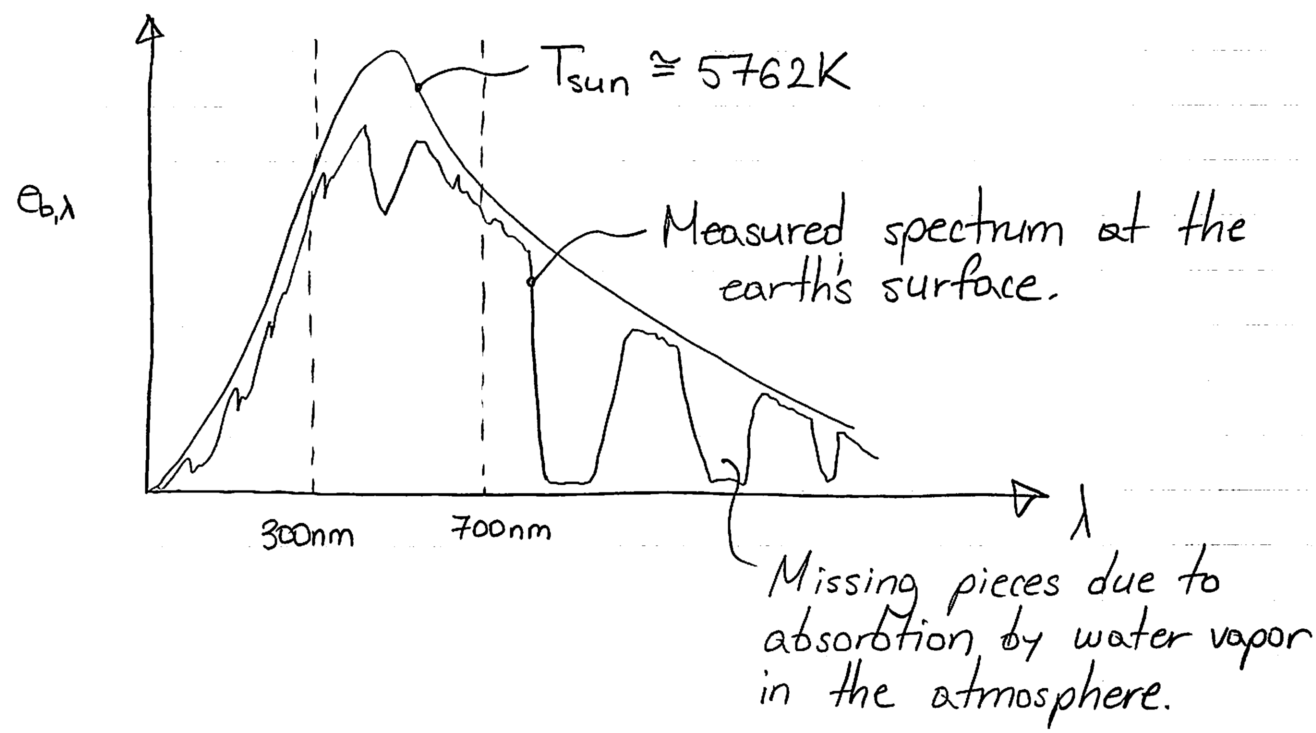
So on a per unit length basis:

$$q'_{r,1} = h(\pi r_o)(T_2 - T_m) + h(2r_o)(T_1 - T_m)$$

$$= 66.2\left[(\pi(0.02m))(696-400) + 2(0.02m)(1000-400)\right]$$

$$\boxed{q'_{r,1} = 2820\ W/m}$$

Solar Radiation



$$q''_{sun} = 1353\ W/m^2 \text{ (in space)}$$

$$q''_{sun,earth} = 636\ W/m^2 \text{ (on the earth's surface)}$$

Note, the total arriving energy from sun to earth is 1.7×10^{14} kW! Peak demand in the US is 1×10^{9} kW!

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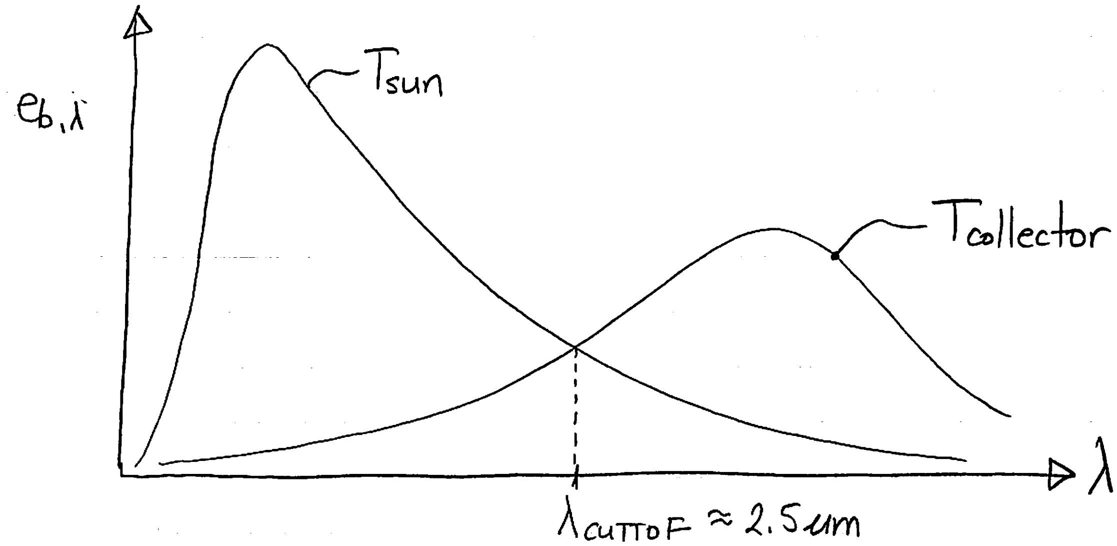
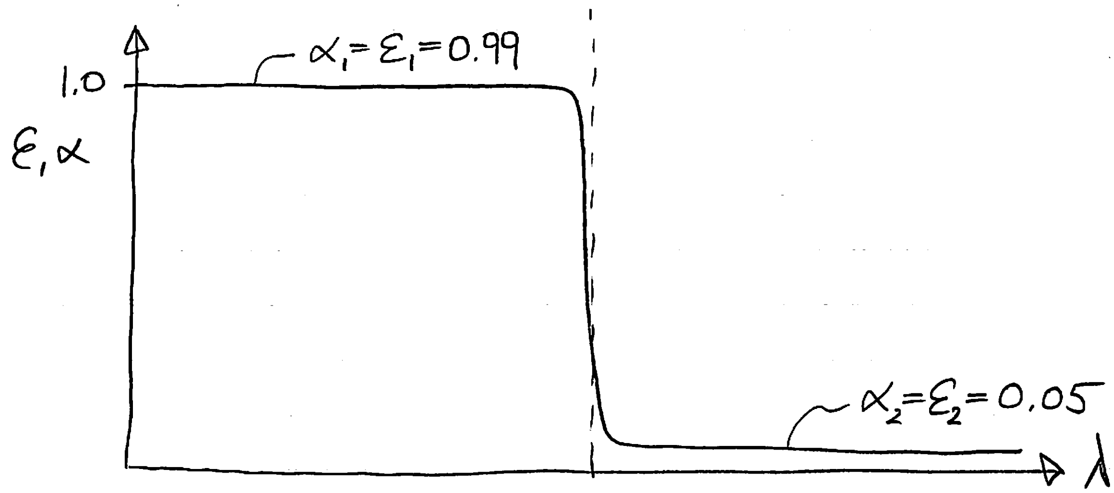
Most solar applications try to absorb some of the heat from the sun.

They use fancy surfaces called selective surfaces:





Because the absorbtion and emission spectra don't overlap, we can simplify our calculations:

$$\boxed{E_{absorbed} = q''_{sun} \cdot \alpha_1 \cdot A}$$; $\alpha_1 \equiv$ absorbtivity in the solar spectrum

$$\boxed{E_{emitted} = \varepsilon_2 A \cdot \sigma T_{surface}^4}$$; $\varepsilon_2 \equiv$ emissivity in the IR spectrum