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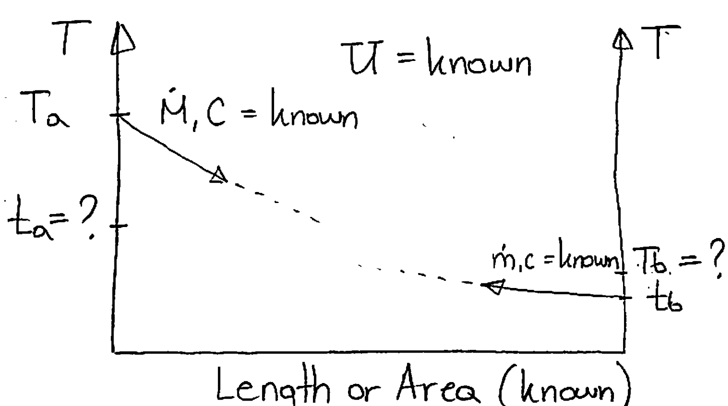
In general, we can say the following:

Parallel Flow	Counter Flow
Disadvantage #1: Large temp. difference at one end of the heat exchanger casuses added thermal stresses & early failure	Advantage #1: More uniform ΔT minimizes the thermal stresses throughout the exchanger.
Disadvantage #2: The outlet temp. of the cold fluid never exceeds the outlet temp of the hot fluid. Less efficient.	Advantage #2: The outlet temp of the cold fluid can approach the highest temperature of the hot fluid More efficient.
	Advantage #3: More uniform ΔT produces a more uniform q.

ε-NTU Method (Effectiveness-NTU method)

Note, in most heat exchanger design problems, we don't know the fluid outlet temperatures, i.e. $T_{1,OUT}$ or $T_{2,OUT}$



Now we can define a quantity called the capacity

$$C_h = (\dot{M}C)_{hot} \; [W/K] \; ; \quad C_c = (\dot{m}c)_{cold} \; [W/K]$$

To solve, we could guess an exit temp., solve for $Q_h = Q_c = C_h \Delta T_h = C_c \Delta T_c$

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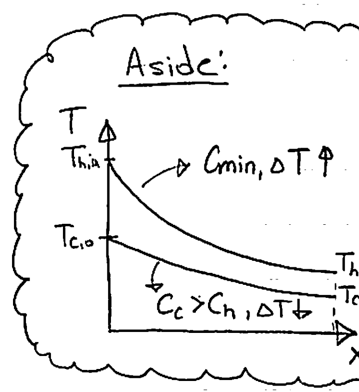
Then we would have to calculate Q from UA LMTD and check against our previous answer. If differing, we would guess another exit temp. and try again.

We can make our lives much easier with ε-NTU.

$$\varepsilon = \frac{\text{actual heat transferred}}{\left(\text{maximum heat that could possibly be transferred from one stream to the other}\right)}$$



Mathematically, this is equal to:

$$\varepsilon = \frac{C_h\left(T_{h,in} - T_{h,OUT}\right)}{C_{min}\left(T_{h,in} - T_{c,in}\right)} = \frac{C_c\left(T_{c,OUT} - T_{c,in}\right)}{C_{min}\left(T_{h,in} - T_{c,in}\right)}$$

where C_{min} is the smaller of C_h & C_c.
The reason we use C_{min} is because the fluid with the lower capacity will be the one which undergoes the maximal temperature change, i.e. $q = C_{min} \Delta T_{max}$, $C_{min} \uparrow, \Delta T \downarrow$

So we can write: $Q = \varepsilon C_{min}\left(T_{h,in} - T_{c,in}\right)$

We can also define: $\boxed{NTU = \frac{UA}{C_{min}}} = \frac{\text{heat rate capacity of h.x.}}{\text{heat capacity rate of flow}}$
(dimensionless)

Using energy balances & simplifying, we can solve for our two cases:

<u>Parallel Flow:</u>

$$-\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right) NTU = \ln\left[-\left(1 + \frac{C_c}{C_h}\right)\varepsilon \frac{C_{min}}{C_c} + 1\right]$$

Solving for ε, we obtain:

$$\varepsilon = \frac{1 - \exp\left[-\left(1 + C_{min}/C_{max}\right) NTU\right]}{1 + C_{min}/C_{max}} = f\left(\frac{C_{min}}{C_{max}}, NTU \text{ only}\right)$$

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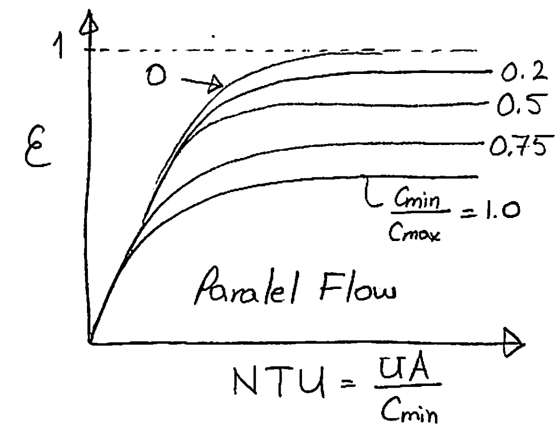
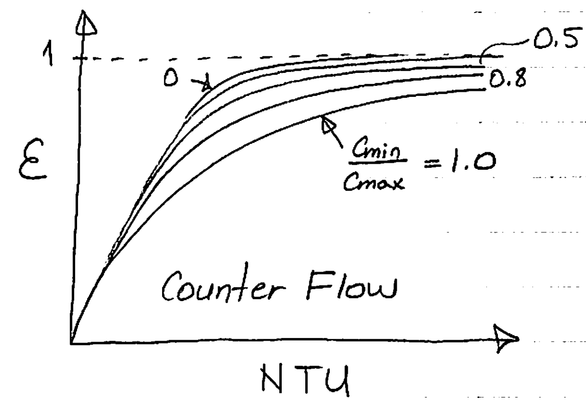
Counter Flow:

$$\varepsilon = \frac{1-\exp\left[-(1+C_{min}/C_{max})NTU\right]}{1-(C_{min}/C_{max})\exp\left[-(1-C_{min}/C_{max})NTU\right]} = f\left(\frac{C_{min}}{C_{max}}, NTU, \text{only}\right)$$

We can plot our results in a more useful form





Note, the ε-NTU method can be applied to any arbitrarily shaped heat exchanger with more difficult flow patterns (i.e. cross flow, mixed flow, etc...)

For a great book, see "Compact Heat Exchangers", Kays & London, (1955.)

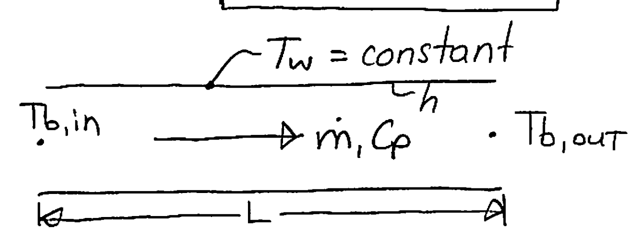
Note that our solution for the pipe flow with constant wall temperature will also simplify to this:

$$\underbrace{\frac{T_{b,out} - T_{b,in}}{T_w - T_{b,in}}}_{\varepsilon} = 1 - e^{\left(\underbrace{-\frac{hPL}{\dot{m}C_p}}\right)} \Rightarrow \text{See page 120 of notes.}$$

$hPL = UA$ $\quad$ P = perimeter

$\dot{m}C_p = C_{min}$

$$\boxed{\varepsilon = 1 - e^{-NTU}} \Rightarrow f(NTU \text{ only})$$



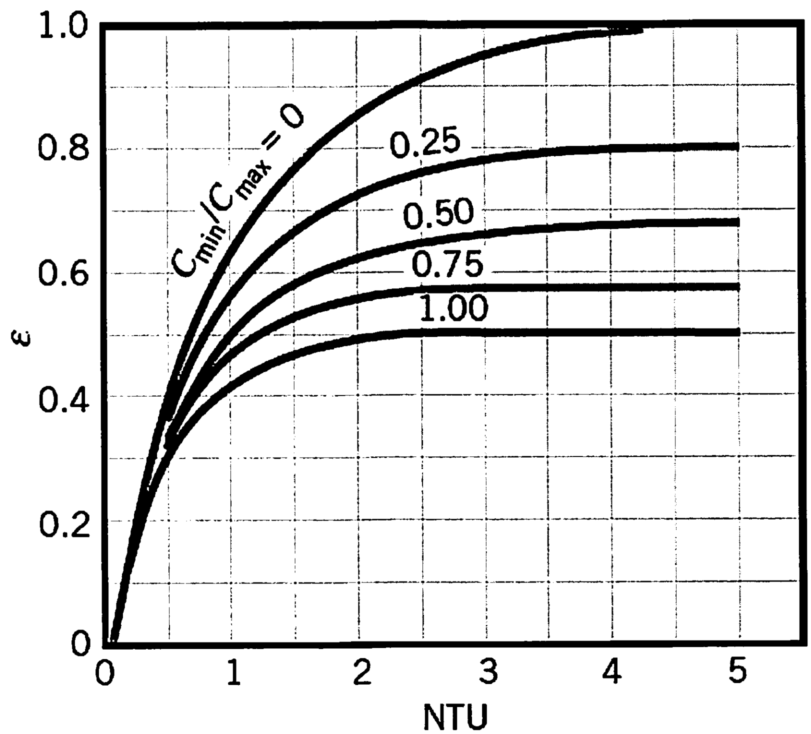


FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

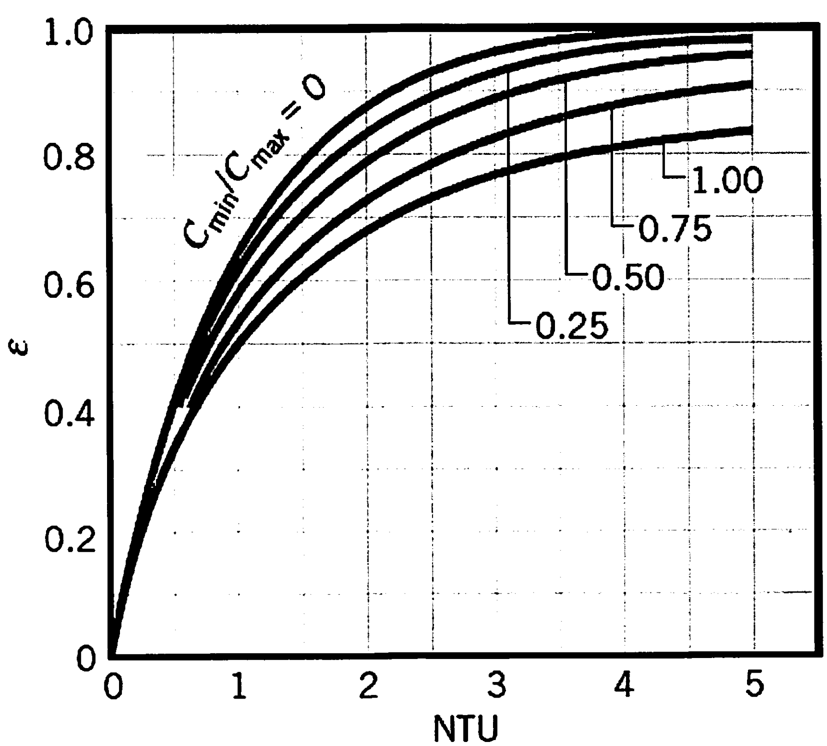


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

Adapted from: Incropera, F.P. and D.P. DeWitt, T.L. Bergman, A.S. Lavine, *Fundamentals of heat and mass transfer*. 6nd ed. 2007, New York: Wiley. Chapter 11, pg. 690.

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Example | Consider the following parallel-flow h.x. specification

Cold flow enters at 40°C, $C_c = 20{,}000\ W/K$
Hot flow enters at 150°C, $C_h = 10{,}000\ W/K$
$A = 30m^2$, $U = 500\ W/m^2 \cdot K$
Determine the heat transfer and the exit temperatures.

Here, we can't use LMTD since we don't have exit conditions so ε-NTU must be used.

$$NTU = \frac{UA}{C_{min}} = \frac{500\,(30)}{10{,}000} = 1.5$$

$$\frac{C_{min}}{C_{max}} = 0.5$$

From our tables, we obtain $\varepsilon = 0.596$

$$Q = \varepsilon C_{min}(T_{h,in} - T_{c,in}) = 0.596\,(10{,}000)(110)$$
$$= 655{,}600\,W = 655.6\,kW$$

From energy balances:

$$T_{h,out} = T_{h,in} - \frac{Q}{C_h} = 150 - \frac{655{,}600}{10{,}000} = 84.44°C$$

$$T_{c,out} = T_{c,in} + \frac{Q}{C_c} = 40 + \frac{655{,}600}{20{,}000} = 72.78°C$$

Example | Suppose we had the same problem, but A = unknown and we want $T_{h,out} = 90°C$. Calculate A.

Here we can use the ε-NTU method or LMTD since we have an exit temperature. Applying an energy balance:

$$T_{c,out} = T_{c,in} + \frac{C_h}{C_c}(T_{h,in} - T_{h,out}) = 40 + \frac{1}{2}(150 - 90) = 70°C$$

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Using the ε-NTU method:

$$\varepsilon = \frac{C_h\left(T_{h,in} - T_{h,out}\right)}{C_{min}\left(T_{h,in} - T_{c,in}\right)} = \frac{10{,}000\,(150-90)}{10{,}000\,(150-40)} = 0.5455$$

From our charts, $NTU \cong 1.15 = UA/C_{min}$

$$A = \frac{10{,}000\,(1.15)}{500} = 23.00\,m^2$$

We could have also used LMTD (note doesn't mater which side is a and b)

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = \frac{\left(T_{h,in} - T_{c,in}\right) - \left(T_{h,out} - T_{c,out}\right)}{\ln\left(\frac{T_{h,in} - T_{c,in}}{T_{h,out} - T_{c,out}}\right)}$$

$$= \frac{(150-40) - (90-70)}{\ln\left(\frac{150-40}{90-70}\right)} = 52.79\,K$$

So from $Q = UA(LMTD)$

$$A = \frac{Q}{U\,(LMTD)} = \frac{10{,}000\,(150-90)}{(500)(52.79)} = 22.73\,m^2$$

The answers differ by 1%, which reflects graph reading innacuracy.

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Example | On page 109 of the notes, we did external flow with variable wall temperature. We solved for:

$$q''|_{y=0} = 0.332\left(\frac{k}{x}\right)Re_x^{1/2}\,Pr^{1/3}\int_0^x \frac{\frac{d(T_w - T_\infty)}{d\xi}}{\left[1-\left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}}\,d\xi$$

Solve for $q''|_{y=0}$ if $(T_w - T_\infty) = \beta\sqrt{x}$

Since it's a continuous temperature change, $\xi \cong x$

$$(T_w - T_\infty) = \beta\sqrt{\xi}$$

$$\frac{d(T_w - T_\infty)}{d\xi} = \frac{1}{2}\beta\frac{1}{\sqrt{\xi}}$$

Let $s = \frac{\xi}{x} \Rightarrow q''|_{y=0} = 0.332\left(\frac{k}{x}\right)Re_x^{1/2}\,Pr^{1/3}\,\beta\cdot\frac{1}{2}\int_0^x \frac{d\xi}{\xi^{1/2}\left[1-\left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}}$

Multiply by $\left(\frac{x}{x}\right)$, we will obtain

$$q''|_{y=0} = 0.332\left(\frac{k}{x}\right)Re_x^{1/2}\,Pr^{1/3}\,x^{1/2}\,\beta\cdot\frac{1}{2}\underbrace{\int_0^1 \frac{ds}{s^{1/2}\left[1-s^{3/4}\right]^{1/3}}}_{\text{We need to solve this}}$$

$$\int_0^1 \frac{ds}{s^{1/2}\left[1-s^{3/4}\right]^{1/3}}, \qquad \begin{aligned} \lambda &= (1-s^{3/4}) \\ s &= (1-\lambda)^{4/3} \\ s^{1/2} &= (1-\lambda)^{2/3} \\ \left[1-s^{3/4}\right]^{1/3} &= \lambda^{1/3} \end{aligned}$$

$$= \frac{4}{3}\int_0^1 \frac{(1-\lambda)^{1/3}\,d\lambda}{(1-\lambda)^{2/3}\,\lambda^{1/3}} = \frac{4}{3}\int_0^1 \frac{d\lambda}{\lambda^{1/3}(1-\lambda)^{1/3}} = \frac{4}{3}\int_0^1 x^{\frac{2}{3}-1}(1-\lambda)^{2/3-1}\,d\lambda$$

So we know from integral tables: $\int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$

$$= \frac{(1.355)^2}{0.893} = 2.06$$

$$q''|_{y=0} = 0.332\left(\frac{k}{x}\right)Re_x^{1/2}\,Pr^{1/3}\,\underbrace{x^{1/2}\,\beta}_{\Delta T(x)}\,\frac{1}{2}\cdot\frac{4}{3}(2.06)$$

$$\boxed{Nu_x = \left(\frac{q''|_{y=0}\cdot x}{\Delta T(x)\cdot k_f}\right) = 0.455\,Re_x^{1/2}\,Pr^{1/3}}$$

$\Rightarrow$ Note, we get constant heat flux only if $T(x) = \beta\sqrt{x}$

END OF LECTURE 14