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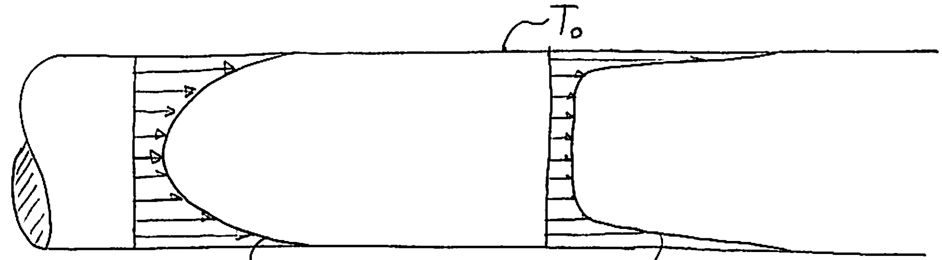
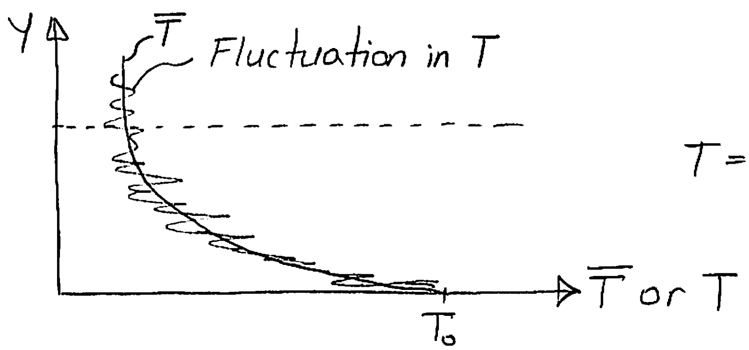
Note however we want to solve for heat transfer



We can follow the exact same procedure as we did with the momentum balance, but now with energy



$$T = \overline{T} + T'$$

$$\overline{q''} = -\overline{k\frac{\partial T}{\partial y}} + \overline{V'T}\rho C_p \Rightarrow \text{Energy integral formulation}$$

$$\overline{q''} = -k\frac{\partial \overline{T}}{\partial y} + \overline{V'T'}\cdot\rho C_p$$

$$\overline{V'T'} = -\varepsilon_T \frac{\partial \overline{T}}{\partial y}$$

Now we can do the following: $Pr_t = \frac{\varepsilon}{\varepsilon_t}$ Turbulent Prandtl Number

So our heat transfer becomes:

$$\boxed{\begin{array}{l} \overline{q''} = -\rho C_p(\alpha + \varepsilon_T)\frac{\partial \overline{T}}{\partial y} \\ \hline \overline{\tau} = \rho(\nu + \varepsilon)\frac{\partial \overline{u}}{\partial y} \end{array}}$$

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We can now make a useful analogy (Turbulent Colburn analogy)

$$\frac{\overline{q''}/\Delta T}{\rho C_p U} = \frac{1}{2}\left(\frac{\overline{\tau_o}}{\frac{1}{2}\rho U^2}\right)\frac{1}{Pr^{2/3}} = \frac{C_f}{2Pr^{2/3}} = \frac{f}{8Pr^{2/3}}$$

Aside: Comes from:

$$\frac{Nu_D}{Re_D \cdot Pr} = \frac{\frac{\bar{h}\cancel{D}}{\cancel{k}}}{\left(\frac{\cancel{\rho} U \cancel{D}}{\cancel{\mu}}\right)(\nu)\left(\frac{C_p \rho}{k}\right)} = \frac{\bar{h}}{\rho C_p U} = \frac{\overline{q''}/\Delta T}{\rho C_p U}$$

$$\boxed{St = \frac{C_f}{2}\cdot\frac{1}{Pr^{2/3}} = \frac{f}{8}\cdot\frac{1}{Pr^{2/3}}} \Rightarrow \text{Stanton number}$$

$$\boxed{St = \frac{Nu_D}{Re_D \cdot Pr} = \frac{\bar{h}}{\rho C_p U}}$$

⇒ Note most results are plotted as St vs Re_D, for experiments.

Note, this is the same as we had before in laminar flow over a flat plate called the Colburn analogy.
See page 90 of the class notes.

$$St \cdot Pr^{2/3} = \frac{C_f}{2} = \frac{f}{8} \quad \text{(For smooth pipes)}$$

$$\frac{Nu_D}{Re_D \cdot Pr}\cdot Pr^{2/3} = \frac{f}{8} \Rightarrow \text{from page 149, } f = 0.184\,Re_D^{-0.2}$$

$$\frac{Nu_D}{Re_D \cdot Pr^{1/3}} = \frac{0.184}{8} Re_D^{-0.2}$$

$$\boxed{Nu_D = 0.023\,Re^{0.8}\,Pr^{1/3}}$$

⇒ Turbulent pipe flow Nusselt # Analytical Result
Valid for: $4\times10^4 < Re_D < 10^5$

Note, the experimental result is:

$$\boxed{Nu_{D,Exp} = 0.023\,Re^{0.8}\,Pr^{0.4}}$$

⇒ Dittus-Boelter equation
⇒ $Re_D > 10\,000$.

Most accurate correlation to use is Gnielinski correlation for Pipe flow

$$\boxed{Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}} \Rightarrow 3000 \le Re_D \le 5\times10^6$$

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Turbulent Heat Transfer over a Flat Plate

For Laminar flow, we derived:

$$C_{f,x} = \frac{\vec{\tau}_{o,x}}{\frac{1}{2}\rho V_\infty^2} = \frac{0.664}{Re_x^{1/2}}$$

; $C_{f,x}$ = skin friction coefficient at location x along plate
$\vec{\tau}_{o,x}$ = shear stress at plate surface at location x along plate
$C_{f,av}$ = average friction coefficient over the whole plate.

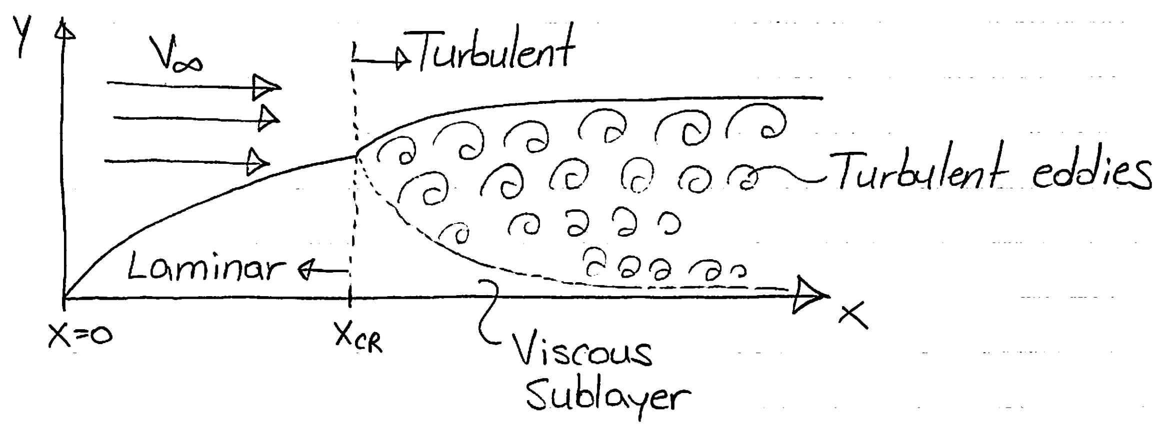
$$C_{f,av} = \frac{1.328}{Re_L^{1/2}}$$

$$\frac{\delta}{x} = \frac{4.92}{Re_x^{1/2}}$$

$$Nu_x = 0.332\, Re_x^{1/2} Pr^{1/3} \Rightarrow \overline{Nu_L} = \frac{\bar{h}L}{k} = 0.664\, Re_L^{1/2} Pr^{1/3}$$

$$St_x \cdot Pr^{2/3} = \frac{C_{fx}}{2}$$

Now we can examine the turbulent flow regime



$$\boxed{Re_{CR} = \frac{V_\infty X_{CR}}{\nu} = 50{,}000 - 500{,}000}$$

Due to the difficulty in analytically modeling the turbulent boundary layer on a flat plate, many of the following expressions stem from experimental correlations.

$$\boxed{\begin{array}{ll} C_{f,x} = 0.0592\, Re_x^{-1/5} & 10^5 \le Re_x \le 10^7 \\ C_{f,x} = 0.026\, Re_x^{-1/7} & 10^7 \le Re_x \le 10^9 \end{array}}$$

$\Rightarrow$ Turbulent flow skin friction coefficient.

For even greater accuracy: (Developed by White)

$$\boxed{C_{f,x} = \frac{0.455}{(\ln 0.06 Re_x)^2} \; ; \quad 10^5 \le Re_x \le 10^9}$$

$\Rightarrow$ Turbulent flow over a flat plate.

Note, x is still the distance from the beginning of the flat plate, not from x_{CR}.

Typically, we want the average skin friction coefficient for the whole plate

$$\vec{\tau}_{o,x} = \frac{1}{2}\rho V_\infty^2 C_{f,x} = \frac{1}{2}\rho V_\infty^2 \cdot 0.0592 Re_x^{-0.2}$$

$$= \frac{1}{2}\rho V_\infty^2 \cdot 0.0592 \left(\frac{\rho V_\infty x}{\mu}\right)^{-0.2}$$

$$= C x^{-0.2} \quad \text{where } C = \frac{1}{2}\rho V_\infty^2 \cdot 0.0592 \left(\frac{\rho V_\infty}{\mu}\right)^{-0.2}$$

$$\vec{\tau}_{av,L} = \frac{C}{L}\int_0^L \frac{dx}{x^{0.2}} = \frac{C x^{0.8}}{0.8L} = \frac{C}{0.8L^{0.2}} = \frac{CL^{-0.2}}{0.8} = \frac{\tau_{o,L}}{0.8}$$

We can now solve for $C_{f,av}$

$$C_{f,av} = \frac{C_{f,L}}{0.8} = \frac{0.0592 Re_L^{-1/5}}{0.8}$$

$$\boxed{C_{f,av} = 0.074 Re_L^{-1/5}}$$

Applying our Colburn analogy: $St_x = \frac{Nu_x}{Re_x \cdot Pr}$ (Stanton number)

We know for a flat plate: $St_x \cdot Pr^{2/3} = \frac{C_{f,x}}{2}$

$$\frac{Nu_x}{Re_x Pr} \cdot Pr^{2/3} = \frac{C_{f,x}}{2}$$

$$Nu_x = \frac{C_{f,x}}{2} Re_x Pr^{1/3}$$

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Using $C_{f,x} = 0.0592\, Re_x^{-1/5}$

$$Nu_x = \frac{0.0592}{2} Re_x^{-1/5} \cdot Re_x Pr^{1/3}$$

$$\boxed{Nu_x = 0.029\, Re_x^{0.8} Pr^{1/3}} \Rightarrow \text{Colburn correlation (from analogy)}$$

$$\boxed{Nu_x = 0.029\, Re_x^{0.8} Pr^{0.43}} \Rightarrow \text{Whitacker correlation}$$

$$\boxed{\overline{Nu}_L = \frac{\bar{h}L}{k_f} = 0.036\, Re_L^{0.8} Pr^{0.43}} \Rightarrow \begin{cases} 0.7 < Pr < 400 \\ 5\times10^5 < Re_x < 3\times10^7 \end{cases}$$

For a more accurate correlation

$$\boxed{Nu_x = \frac{\left(\frac{C_{f,x}}{2}\right) \cdot Re_x Pr}{1 + 12.7\left(\frac{C_{f,x}}{x}\right)^{1/2}\left(Pr^{2/3} - 1\right)}} \Rightarrow \text{White correlation}$$

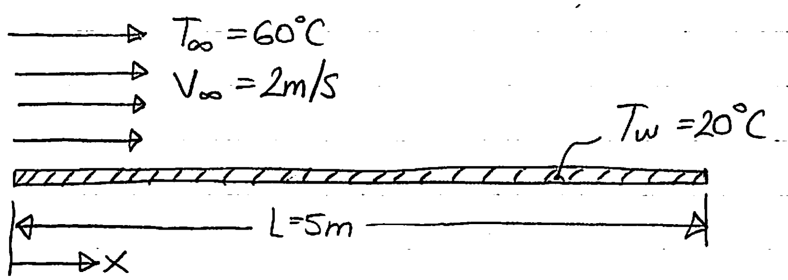
$0.5 < Pr < 2000$

$5\times10^5 < Re_x < 10^7$

Note, the closest we got to an analytical result was with the Colburn analogy, but even then we had to use the local skin friction coefficient ($C_{f,x}$) based on experiments.

For more reading, see page 312 in Mills textbook.

Example) Engine oil at 60°C flows over a 5m long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the total drag force and heat transfer per unit width of the entire plate.



Assume : 1) $T_w = \text{constant} \neq f(x) = 20°C$
2) Constant properties, steady flow.

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$$T_f = \overline{T} = \frac{T_w + T_\infty}{2} = 40°C$$

$\rho_{oil} = 876\ kg/m^3$
$k_{oil} = 0.144\ W/m \cdot K$
$Pr = 2870$ (Note how large this Prandtl number is!)
$\nu_{oil} = 242 \times 10^{-6}\ m^2/s$

$Pr_{water} \approx 7$
$Pr_{air} \approx 1$

First we check if we are in the turbulent or laminar regime

$$Re_L = \frac{V_\infty L}{\nu_{oil}} = 4.13 \times 10^4 < 5.0 \times 10^5 \text{ (Laminar flow)}$$

We know from our previous derivations (page 85 of notes)

$$C_{f,av} = \frac{\tau_{o,av}}{\frac{1}{2}\rho V_\infty^2} = 1.328\, Re_L^{-0.5} = 0.00653$$

$$F_D = C_{f,av} \cdot A \cdot \frac{\rho V_\infty^2}{2} = (0.00653)(5m \cdot 1m)\frac{(876 kg/m^3)(2m/s)^2}{2}$$

$$\boxed{F_D = 57.2\,N}$$

For heat transfer:

$$\overline{Nu}_L = \frac{\overline{h} L}{k_{oil}} = 0.664\, Re_L^{1/2} Pr^{1/3} = 1918$$

$$\overline{h} = 55.2\ W/m^2 \cdot K$$

$$\boxed{Q = hA(T_\infty - T_w) = 11.04\,kW}$$

Now what if we 15x the speed of our flow? Will our friction and heat transfer 15x as well?

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$$Re_L = \frac{V_\infty L}{\nu_{oil}} = 6.2 \times 10^5 > Re_{crit} \quad \text{(Turbulent flow)}$$

$$C_{f,av} = 0.074\, Re_L^{-0.2} = 0.00514 \quad \text{(page 153 of notes)}$$

$$F_D = C_{f,av} \cdot A \cdot \frac{\rho V_\infty^2}{2} = (0.00514)(5m^2)\frac{(876\,kg/m^3)(30\,m/s)^2}{2}$$

$$\boxed{F_D = 10.13\,kN}$$

For heat transfer:

$$\overline{Nu_L} = \frac{\bar{h}L}{k_{oil}} = 0.036\, Re_L^{0.8} Pr^{0.43} \quad \text{(Estimate only since } Pr > 400)$$

↳ Could have used White correlation

$$= 0.036\,(6.2 \times 10^5)^{0.8}(2870)^{0.43} = 47\,545$$

$$\bar{h} = \frac{\overline{Nu_L} \cdot k_{oil}}{L} = \frac{(47\,545)(0.144\,W/m \cdot K)}{5m}$$

$$\bar{h} = 1369.3\,W/m^2 \cdot K$$

$$\boxed{Q = \bar{h}A(T_\infty - T_w) = 273.9\,kW}$$

Now lets compare:

$$\frac{V_{\infty,2}}{V_{\infty,1}} = \frac{15}{1} = 15$$

$$\frac{F_{D,2}}{F_{D,1}} = \frac{10\,130\,N}{57.2\,N} = 177.1 \quad \text{(Drag increases dramatically)}$$

$$\frac{Q_2}{Q_1} = \frac{273.9\,kW}{11.04\,kW} = 24.8 \quad \text{(Heat transfer increases modestly)}$$

Note, this is where you strike a balance in heat exchanger design in terms of heat transfer and pressure drop.

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Note also, another important factor to keep in mind is the working fluid. In this case, oil is used with a high Prandtl number and high viscosity. If we had chosen water or air, how would our results change:

Water: $\rho_w = 1000\,kg/m^3$
$k_w = 0.6\,W/m\cdot K$
$Pr = 7$
$\nu_w = 0.658\times10^{-6}\,m^2/s$

$$Re_L = \frac{V_\infty L}{\nu_w} = \frac{(2m/s)(5m)}{0.658\times10^{-6}m^2/s} = 1.52\times10^7 \text{ (Turbulent flow)}$$

$$C_{f,av} = 0.074\,Re_L^{-0.2} = 0.0027$$

$$F_{D,w} = C_{f,av}\cdot A\cdot\frac{\rho V_\infty^2}{2} = (0.0027)(5m^2)\frac{(1000kg/m^3)(2m/s)^2}{2}$$

$$\boxed{F_{D,w} = 27.1\,N}$$

For heat transfer:

$$\overline{Nu_L} = \frac{\bar{h}L}{k_w} = 0.036\,Re_L^{0.8}\,Pr^{0.43}$$

$$= 0.036\,(1.52\times10^7)^{0.8}(7)^{0.43} = 46256.2$$

$$\bar{h} = \frac{\overline{Nu_L}\cdot k_w}{L} = \frac{(46256.2)(0.6W/m\cdot K)}{5m}$$

$$\bar{h} = 5550.8\,W/m^2\cdot K$$

$$\boxed{Q_w = \bar{h}A(T_\infty - T_w) = 1.11\,MW}$$

$$\frac{F_{D,oil}}{F_{D,w}} = 2.11 \qquad \frac{Q_{oil}}{Q_w} = 0.01 \text{ or } 1\% \text{ only.}$$

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This simple analysis shows the importance of selecting the proper working fluid. You may be asking yourself, why would anybody use a high Pr oil as a working fluid in a thermal application? Isn't water always better?

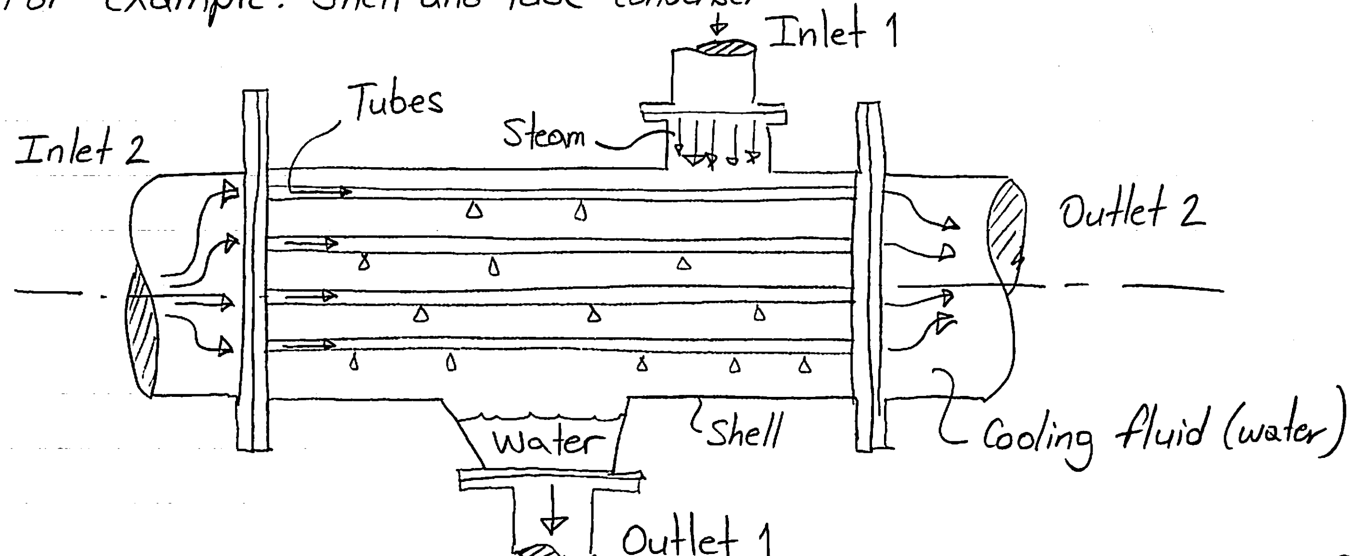
The answer is NO! Oils have 2 main advantages over water.

1) Temperature range (Typically -40 to 300°C). Water will freezo or boil unless its pressurized which makes the advantages of water not worth it (pressurization is tricky and requires a lot of design, maintenence, and for water, high pressures.
2) Low volatility. Water has a relatively high vapor pressure meaning it evaporates and coolant can be lost and needs to be replaced. Oil's vapor pressure at $T = 0\text{-}60°C$ is usually 0 Pa, meaning no evaporative mass loss.
3) Corrosion resistance and enhanced lubrication.

<u>Cylinders and Spheres</u>

So far, we've only looked at internal flow in a tube, and external flow on a flat plate. Most heat exchangers are not in this configuration.

For example: Shell and tube condenser



END OF LECTURE 18