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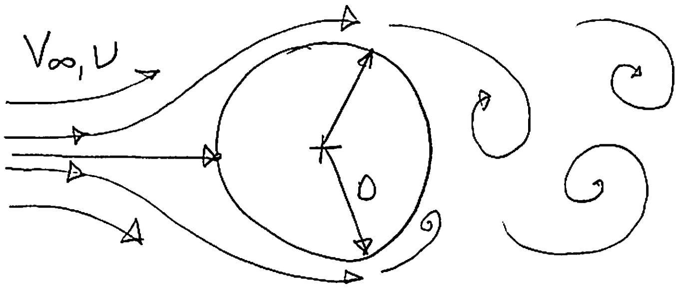
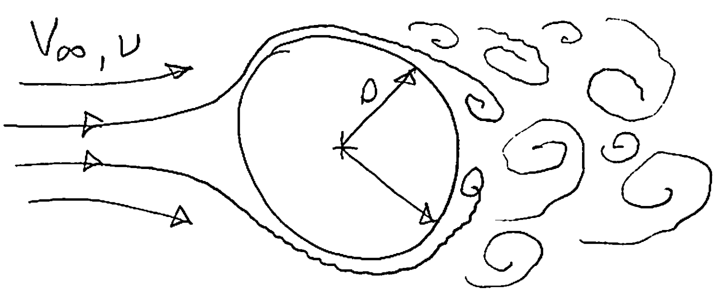
Here we have a tube geometry with external heat transfer.

Since external flow, the turbulence criteria still remains, i.e. $Re_{tr} \approx 3 \times 10^5$.

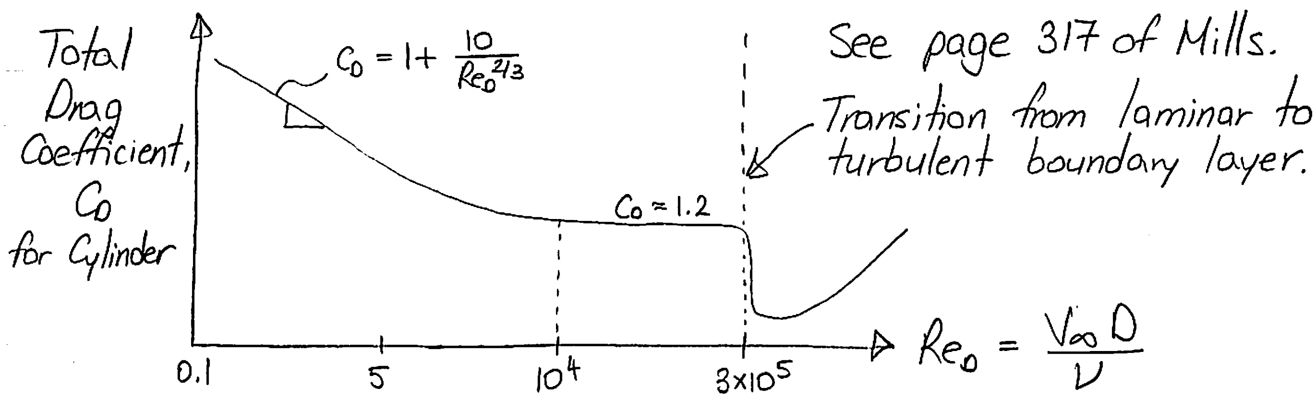


Note, Laminar flow with separation of the laminar boundary layer.
$40 \leq Re_D \leq 150$



Turbulent boundary layer with separation.
$Re_D \geq 3.0 \times 10^5$

In these flows, more criteria is required to describe the flow: laminar or turbulent b.l., separated or intact flow, type of wake. In general:



See page 317 of Mills.

Note, the turbulent boundary layer, although creating more skin friction drag ($C_{f,x}$) decreases the form, pressure, or wake by keeping the boundary layer from separating, due to its higher efficiency mixing and larger momentum.

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$$\boxed{C_D = \frac{F}{\frac{1}{2}\rho V_\infty^2 A_f}}$$, A_f = area of cylinder normal to the flow = $D \cdot L$ (Diameter times length).
F = total drag force due to form and skin frict.

At low Reynolds numbers, $Re_D < 10^4$ (viscous drag dominates)

$$\boxed{C_D = 1 + \frac{10}{Re_D^{2/3}}} \; ; \quad 1 < Re_D < 10^4 \quad \text{(Laminar flow)}$$

$$\boxed{C_D = 1.2} \; ; \quad Re_D > 10^4 \quad \text{(Laminar flow, separated b.l. where wake drag now dominates)}$$

Due to the difficulty in modelling of this flow, most results are from experimental correlations:

$$\boxed{\overline{Nu}_D = \frac{1}{0.8237 - \ln(Re_D Pr)^{1/2}}} \; ; \quad Re_D Pr < 0.2$$

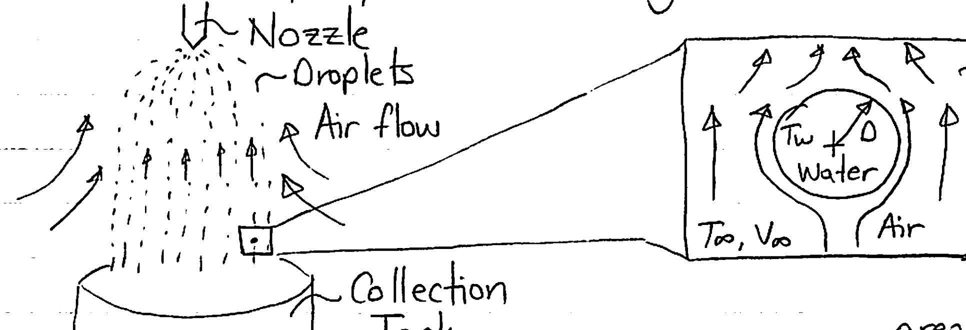
$$\boxed{\overline{Nu}_D = 0.3 + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}} \; ; \quad Re_D < 10^4$$

See pg. 319, equations 4.71b and 4.71c for rest of the regiemes, in Mills.

Flow over a Sphere

The spherical geometry is not a typical heat exchanger geometry per say, however it is vitally important in flows involving droplets and bubbles.

For example: Spray down cooling in a air cooled condenser



Air flow past hot droplet cools it. Very efficient method of cooling since small drops have high surface area/volume ratio.

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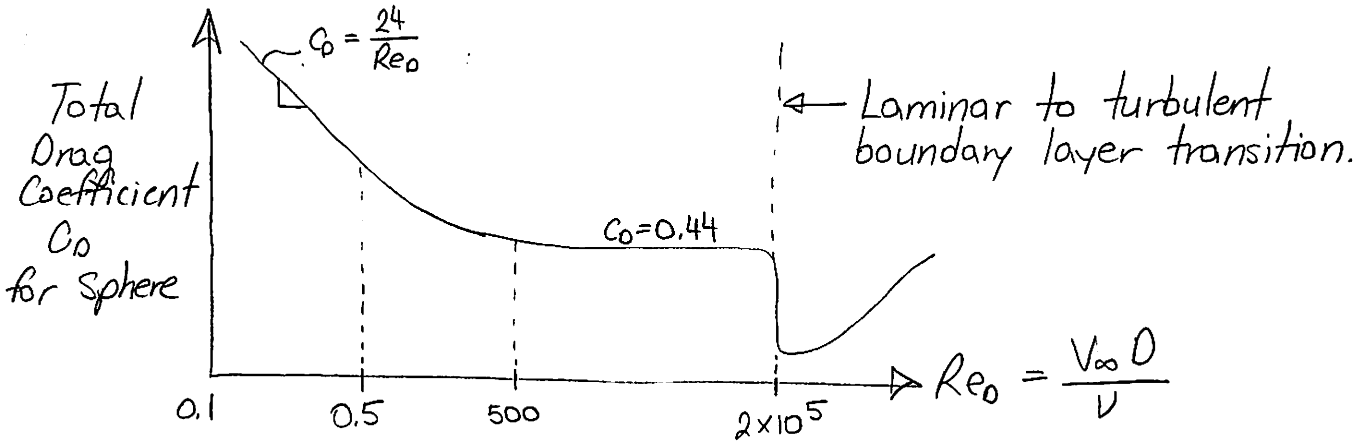

In general, for a sphere:

$$\boxed{C_D = \frac{24}{Re_D}} \; ; \quad Re_D < 0.5 \quad \text{(Laminar flow, Stokes flow)}$$

↳ Can solve for this analytically using our tools.

$$\boxed{F_D = 3\pi\mu V_\infty D} \Rightarrow \text{Stokes drag}$$

$$\boxed{C_D \cong \frac{24}{Re_D}\left(1 + \frac{Re_D^{2/3}}{6}\right)} \; ; \quad 2 < Re_D < 500$$

$$\boxed{C_D \approx 0.44} \; ; \quad 500 < Re_D < 2\times10^5$$

Experimental Correlations

For heat transfer:

$$\boxed{\overline{Nu_D} = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4}} \; ; \quad \begin{matrix} 3.5 < Re_D < 8\times10^4 \\ 0.7 < Pr < 380 \end{matrix}$$

Note, as $Re_D \to 0$, $\overline{Nu_D} = 2$ which corresponds to a sphere conducting heat to an infinite medium:

$$Q_{sph} = \frac{4\pi k (T_1 - T_2)}{1/R_1 - 1/R_2} = \frac{\Delta T}{R_{Thermal}}$$

$$D = 2R_1 \; , \; R_2 \to \infty$$

$$Q_{sph} = \frac{2\cancel{4}\pi k \Delta T D}{\cancel{2}} = 2\pi k D \Delta T$$

$$\frac{1}{A_{sph}}\left(\frac{Q_{sph}}{\Delta T}\right)\cdot\frac{D}{k} = \frac{1}{2\cancel{4}\cancel{\pi}R^2}\cdot 2\cancel{\pi}\cancel{k}D\cdot\frac{D}{\cancel{k}} = \frac{4R^2}{2R^2} = 2 \Rightarrow \boxed{Nu_D = 2} \Rightarrow \text{Makes sense.}$$

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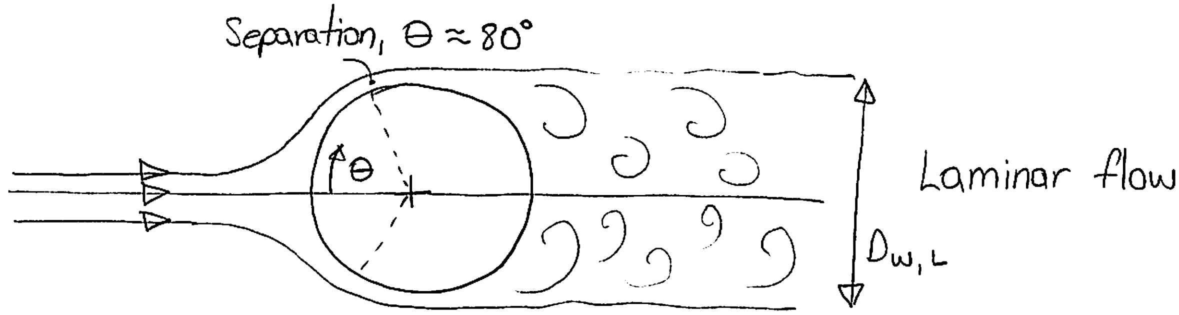
Note, the drag on a bluff body can be decomposed into two components:

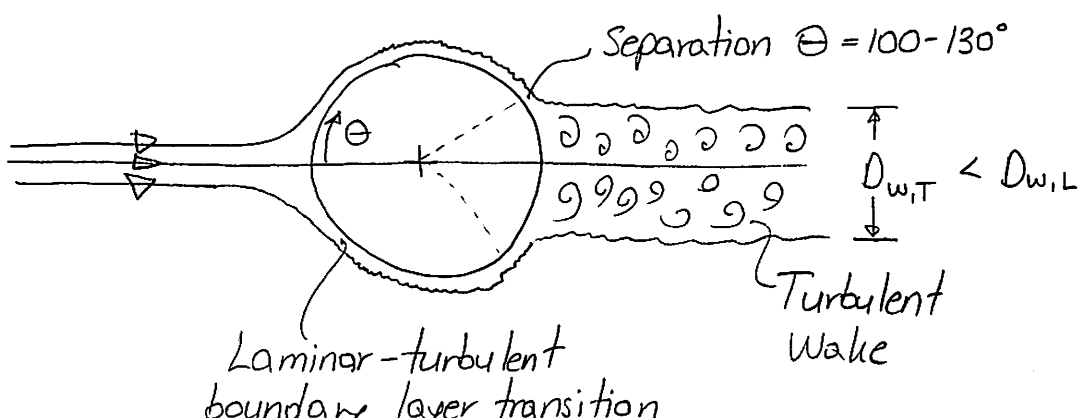
$$F_D = F_{SF} + F_{FD}$$

F_{FD} ← Form drag due to pressure difference or wake drag.

F_{SF} → Skin Friction Drag, $C_{f,x}$ or Viscous drag.

For flow past a sphere or cylinder, turbulence inside the boundary layer plays a very important effect on drag as we've already seen in the C_D plots:





The creation of a turbulent boundary layer tends to enhance mixing, increase momentum in opposition to adverse pressure gradients, and keep the boundary layer attached for longer, reducing the form drag (F_{FD}) more than increasing the skin friction drag (F_{SF}). Note, this is why golf balls are dimpled!

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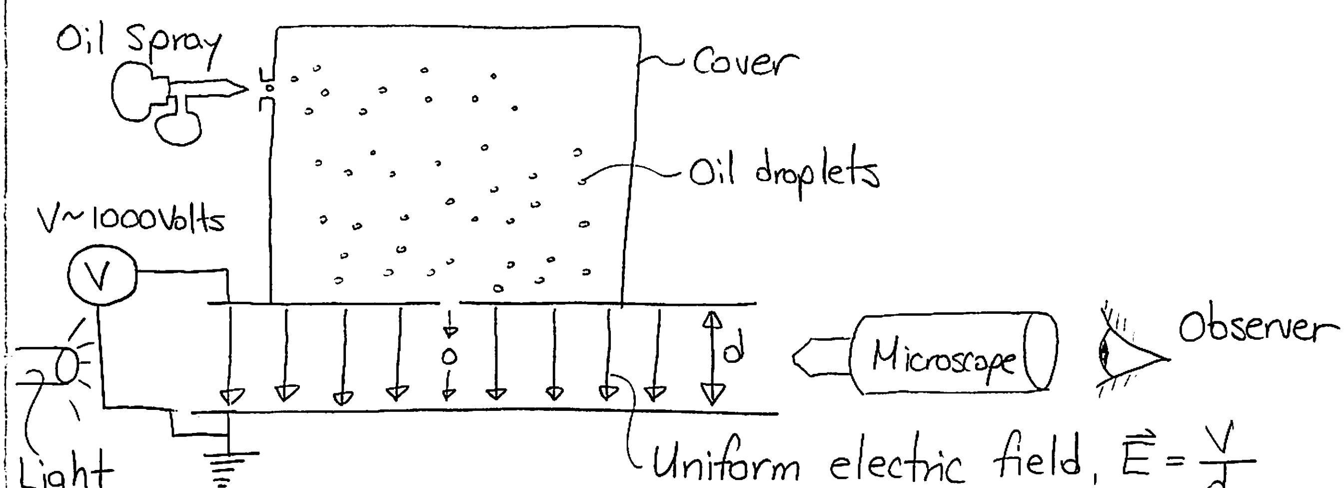
Example | Stokes flow and charge of an electron

In the early 1900's, scientists thought that electical charge wasn't quantized and was continuous. Robert A. Millikan and his graduate student Fletcher wanted to check.

The experiment:



Millikan and Fletcher would spray oil droplets and let one settle into the gap between the plates.

They used oil due to its low vapor pressure (low evaporation) and because it would gain some charge when sprayed. (tribo-electrification)

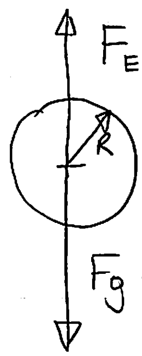
The analysis:

Millikan would first let the droplet fall with no applied electric field and measure its terminal velocity V_t.

Assuming that $Re_0 \ll 1$, he used Stokes drag to calculate the droplet size:

$$F_D = 6\pi\mu V_t R = F_g = \frac{4}{3}\pi R^3 g(\rho_{oil} - \rho_{air}) \Rightarrow \boxed{R = \sqrt{\frac{9\mu_{oil} V_t}{2g(\rho_{oil} - \rho_{air})}}}$$

Next he would turn the electric field on just enough so that the electric field force was just high enough to balance the gravitational force. So the droplet would remain perfectly still in the microscope:



$$F_E = q \cdot E = q \cdot \frac{V}{d}$$

$\Rightarrow$ Single droplet that he focused on between the parallel plates.

$$F_g = \frac{4}{3}\pi R^3 g(\rho_{oil} - \rho_{air})$$

$$q \cdot \frac{V}{d} = \frac{4}{3}\pi R^3 g(\rho_{oil} - \rho_{air})$$

$$q = \frac{4}{3}\frac{d}{V}\pi R^3 g(\rho_{oil} - \rho_{air})$$

$$= \frac{4}{3}\frac{d}{V}\cdot \pi g(\rho_{oil} - \rho_{air})\left(\frac{9\mu_{oil} V_t}{2g(\rho_{oil} - \rho_{air})}\right)^{3/2}$$

$$\boxed{q = 12.7279\pi \frac{(\mu_{oil} V_t)^{3/2}}{(g(\rho_{oil} - \rho_{air}))^{1/2}} \cdot \frac{d}{V}}$$ $\Rightarrow$ electric charge on the individual droplet.

$\hookrightarrow$ Measured parameters: V_t, V, d, ρ_{oil}, ρ_{air}, μ_{oil}

He then measured the charge of many droplets and tabulated them. From his results, Millikan showed a fascinating trend.

$$q_{droplet} = n \cdot q_{e^-}, \text{ where } \boxed{q_{e^-} = 1.5924 \times 10^{-19} \text{ Coulombs}}$$

Todays accepted value is: $\boxed{q_{e^-} = 1.602176 \times 10^{-19} \text{ Coulombs}}$

Millikan showed definitively that electric harge was indeed quantized and won the Nobel prize in 1923 for it. The main tool he used was Stokes' drag!

END OF LECTURE 19