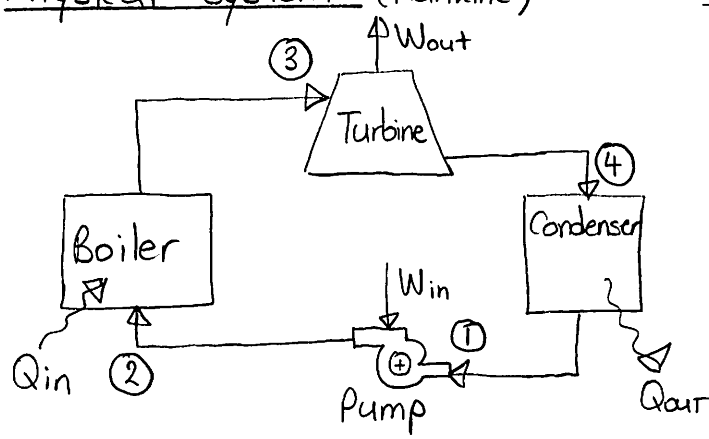
Nenad Miljkovic

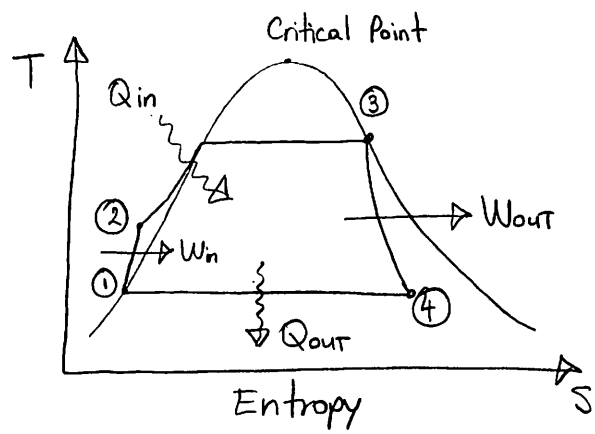
Condensation Heat Transfer

Why do we care? Well 90% of our electricity generated comes from steam power plants. In these plants, the Rankine cycle is used which requires a condenser. Increasing this condensation heat transfer efficiency can increase the overall power plant efficiency and help reduce CO_2 emissions, fuel consumption and cost.

Physical System: (Rankine)

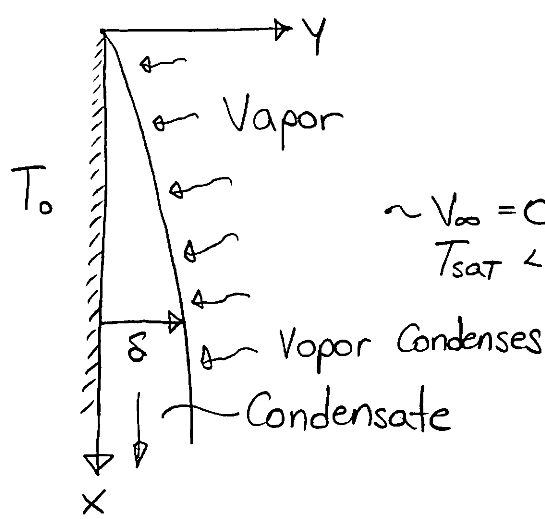


Thermodynamic States:



The condensation and boiling processes are very important for the efficient operation of our most common means of energy production.

Condensation on a Vertical Flat Plate



Assumptions:

1) Pure vapor
2) Vapor is stationary ($V_\infty = 0$)
3) $\tau = 0$ at the l-v interface
4) Inertia can be neglected or Laminar film condensation.
5) Constant properties
6) $\frac{\partial p}{\partial y} = 0$ (pressure gradients in the y-direction are negligible)

Nenad Miljkovic

Note assumption #4 is valid since the latent heat of fluids is typically much larger than their specific heat. Therefore we will have very thin films during filmwise condensation. These films move very slowly (due to high shear at the wall) and inertia can be neglected when compared to viscous or gravity forces.

Aside:
Another way to show inertia ≈ 0 is conservat of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$v = 0, \; \frac{\partial v}{\partial y} = 0$$

So $\frac{\partial u}{\partial x} = 0$

x-momentum: (in the condensate)

$$u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{=0} + \underbrace{\cancel{v}}_{v=0}\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial P}{\partial x} + \nu_f \frac{\partial^2 u}{\partial y^2} + g$$

$$\frac{1}{\rho_f}\frac{\partial P}{\partial x} = \nu_f \frac{\partial^2 u}{\partial y^2} + g \quad (1)$$

x-momentum: (in the vapor)

$$u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{0} + v\underbrace{\cancel{\frac{\partial u}{\partial y}}}_{0} = -\frac{1}{\rho_g}\frac{\partial P}{\partial x} + \underbrace{\nu \frac{\partial^2 u}{\partial y^2}}_{0\,(V_\infty = 0)} + g$$

Since $V_\infty = u = v = 0$

$$\frac{\partial P}{\partial x} = \rho_g g \quad (2)$$

$\Rightarrow$ Hydrostatic pressure (something similar on pg. 131 of notes)

Substituting (2) into (1), we obtain:

$$\frac{1}{\rho_f} \cdot \rho_g g = \nu_f \frac{\partial^2 u}{\partial y^2} + g$$

$$-g(\rho_f - \rho_g) = \mu_f \frac{\partial^2 u}{\partial y^2} \quad \Rightarrow \text{Multiply both sides by } \frac{1}{\rho_f}$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\rho_f - \rho_g}{\rho_f} \cdot \frac{g}{\nu_f} \quad (3)$$

Now we can look at heat transfer. Since inertia is negligible, we can neglect the convective terms in the energy equation. We are left with simple conduction across the film.

Nenad Miljkovic

$$\underbrace{u\frac{\partial T}{\partial x}}_{\text{0 since inertia is negligible}} + \cancelto{v=0}{v\frac{\partial T}{\partial y}} = \alpha\frac{\partial^2 T}{\partial y^2} \Rightarrow \underbrace{\alpha\frac{\partial^2 T}{\partial y^2}}_{\text{Only in y-dir since gradient is in y.}} = 0 \quad (4)$$

Now we can apply our B.C.'s and solve equation (3)

$u(y=0) = 0$ (No slip at the wall)

$\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = 0$ (No shear stress at the l-v interface since $\frac{\mu_g}{\mu_f} \ll 1$)

Solving, we obtain:

$$u = \frac{(\rho_f - \rho_g) g \delta^2}{2\mu_f}\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] \quad (5)$$

Applying our B.C.'s to our energy equation (4)

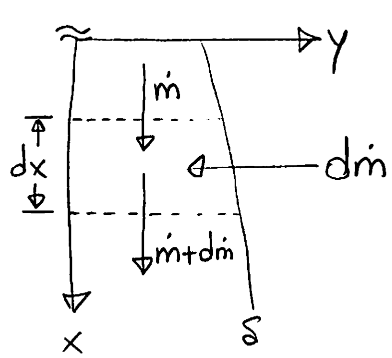
$T(y=0) = T_o$

$T(y=\delta) = T_{sat}$

Solving, we obtain:

$$T = T_o + (T_{sat} - T_o)\frac{y}{\delta} \quad (6)$$

Note that both (5) and (6) are in terms of δ. We need to solve for δ if we want to move on. Looking closely at our condensate.



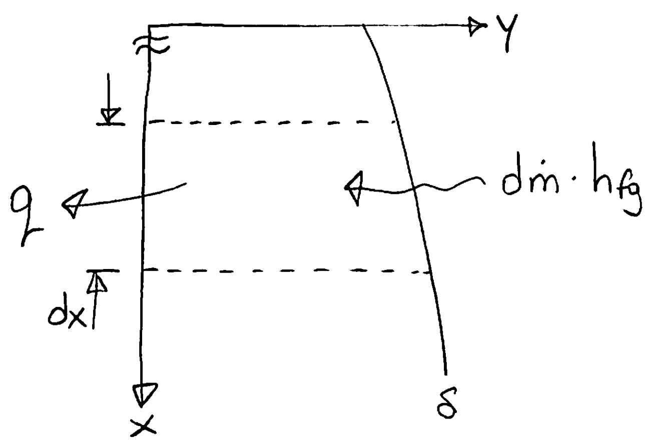
$$\dot{m} = \int_0^\delta \rho_f u\, dy = \int_0^\delta \frac{\rho_f(\rho_f - \rho_g) g \delta^2}{2\mu_f}\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] dy$$

Nenad Miljkovic

$$= \frac{\rho_f(\rho_f-\rho_g)g\delta^2}{2\mu_f}\underbrace{\left[\frac{y^2}{\delta}-\frac{y^3}{3\delta^2}\right]\Bigg|_0^{\delta}}_{\frac{\delta^{\cancel{2}}}{\cancel{\delta}}-\frac{\delta^{\cancel{3}}}{3\cancel{\delta^2}}=\frac{2}{3}\delta}$$

$$\dot{m} = \frac{\rho_f(\rho_f-\rho_g)g\delta^3}{3\mu_f} \quad (7) \Rightarrow \text{Per unit width of plate } \left[\frac{kg}{m\cdot s}\right]$$

We can do the exact same procedure in terms of energy:



h_{fg} = latent heat of phase change [J/kg]
(How much heat is released from converting 1 kg of vapor into liquid).

Doing an energy balance

need to divide out by dx since q'' is a flux, and $d\dot{m}\cdot h_{fg}$ is not.

$$q'' = k\frac{\partial T}{\partial y}\bigg|_{y=0} = d\dot{m}\cdot h_{fg}\left(\frac{1}{dx}\right)$$

From equation (6), $\frac{\partial T}{\partial y} = \frac{T_{sat}-T_0}{\delta}$

$$k\frac{T_{sat}-T_0}{\delta} = h_{fg}\frac{d\dot{m}}{dx} \quad (8)$$

We know from (7) to solve for $\frac{d\dot{m}}{dx}$:

$$k\frac{T_{sat}-T_0}{\delta} = \frac{h_{fg}\rho_f(\rho_f-\rho_g)}{\mu}g\delta^2\frac{\partial\delta}{\partial x}$$

$$\int_0^x \frac{k(T_{sat}-T_0)\mu}{h_{fg}\rho_f(\rho_f-\rho_g)g}\cdot dx = \int_0^{\delta(x)}\delta^3 d\delta = \frac{\delta^4}{4}$$

$\delta(x=0)=0$
$\delta(x)=\delta$

$$\delta = \left[\frac{4k_f(T_{SAT}-T_o)\mu_f x}{\rho_f(\rho_f-\rho_g)g h_{fg}}\right]^{1/4}$$

Now we can solve for our Nusselt number:

$$h = \frac{q''}{T_{SAT}-T_o} = \frac{1}{T_{SAT}-T_o}\left[\frac{k_f(T_{SAT}-T_o)}{\delta}\right] = \frac{k_f}{\delta}$$

$$Nu_x = \frac{hx}{k_f} = \frac{\cancel{k_f}}{\delta}\cdot\frac{x}{\cancel{k_f}} = \frac{x}{\delta}$$

$$\boxed{Nu_x = 0.707\left[\frac{\rho_f(\rho_f-\rho_g)g h_{fg} x^3}{\mu_f k_f(T_{SAT}-T_o)}\right]^{1/4}}$$

$\Rightarrow$ Laminar filmwise condensation Nusselt #.

Note, this analysis was first done by Nusselt in 1916 and is still widely used and accuratelly validated.

For average Nusselt number over the whole plate:

$$\bar{h} = \frac{1}{L}\int_0^L h(x)dx = \frac{4}{3}h(L) \Rightarrow \overline{Nu_L} = \frac{\bar{h}L}{k_f}$$

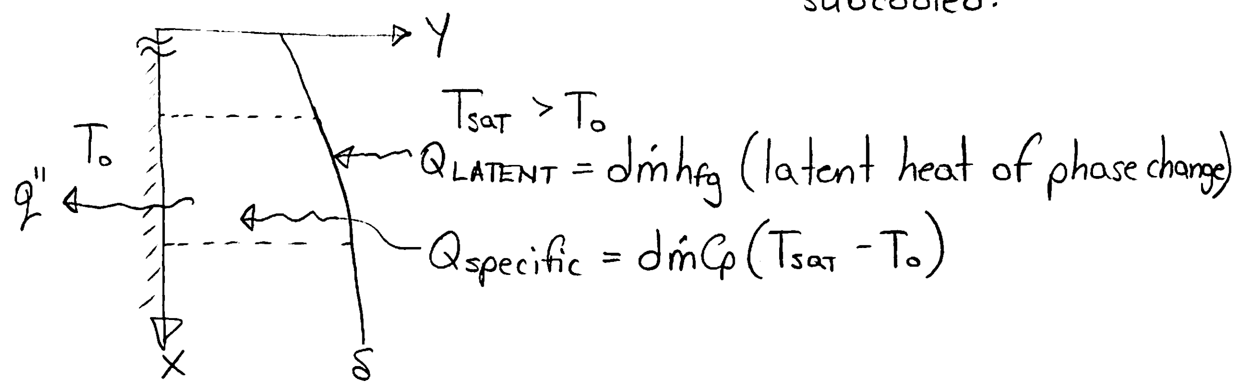
$$\boxed{\overline{Nu_L} = 0.943\left[\frac{\rho_f(\rho_f-\rho_g)g h_{fg} L^3}{\mu_f k_f(T_{SAT}-T_o)}\right]^{1/4}}$$

Some Notes:

1) Liquid condensate properties (μ_f, ρ_f, k_f) are evaluated at the film temperature: $\bar{T} = (T_{SAT} + T_o)/2$.
2) ρ_g and h_{fg} are evatuated at T_{SAT}.
3) In reality, film condensation depends on the specific or sensible heat absorbed by the condensate in addition to the latent heat (which we've considered)

Nenad Miljkovic

Since $T_{sat} > T_w$, there is heat transfer due to sensible heat, i.e. the condensate is not always at T_{sat}, it gets subcooled.



To correct for sensible heating effects, Nusselt and Rohsenow suggested to use a modified h_{fg}.

$$\boxed{h_{fg}' = h_{fg} + 0.68\, C_{p,f} (T_{sat} - T_o)}$$

This sensible heating effect can be characterized by the Jacob number, Ja, to honor Max Jacob's pioneering work of the 1930's on phase change.

$$\boxed{Ja = \frac{\text{sensible heat absorbed}}{\text{latent heat absorbed}} = \frac{C_{p,f} (T_{sat} - T_o)}{h_{fg}}}$$

For water at 1 atm ($T_{sat} = 100°C$) and $T_o = 90°C$, then

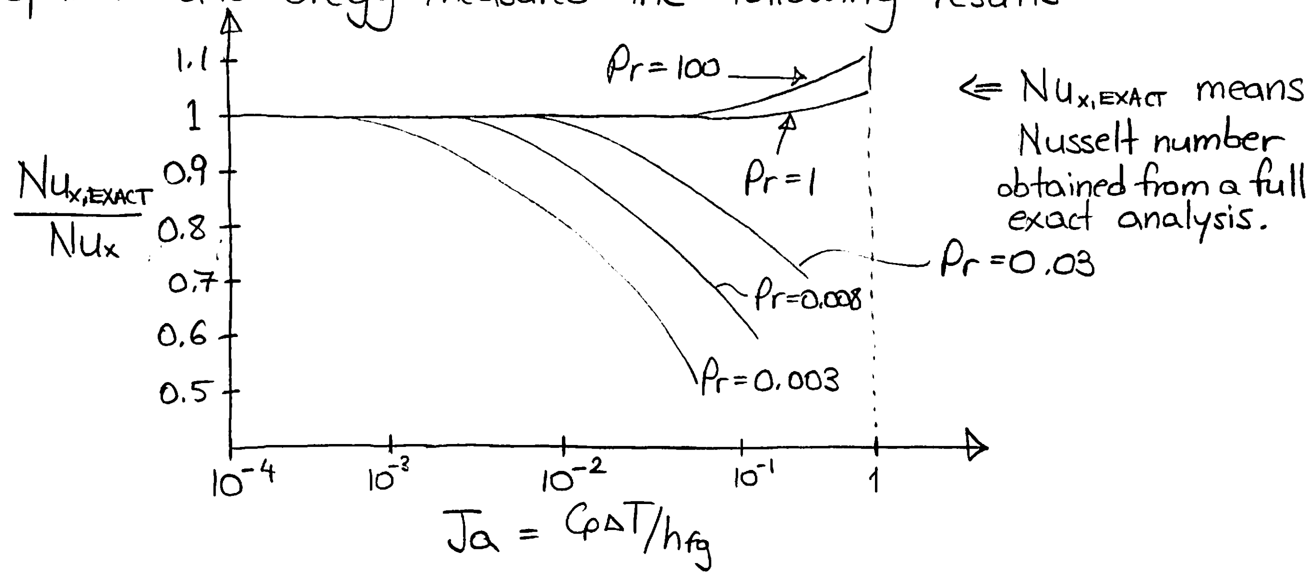
$$Ja = \frac{(4.174\ kg/kg\cdot K)(10K)}{2257\ kJ/kg} = 0.0185$$

So sensible heating only makes up about 2% of the total heat transfer, making its neglect in the previous analysis appropriate.

4) Prandtl number plays a large role as well, exspecially for $Pr \ll 1$. If $Pr > 1$, the Nusselt analysis is valid.

Nenad Miljkovic

Sparrow and Gregg measured the following results:

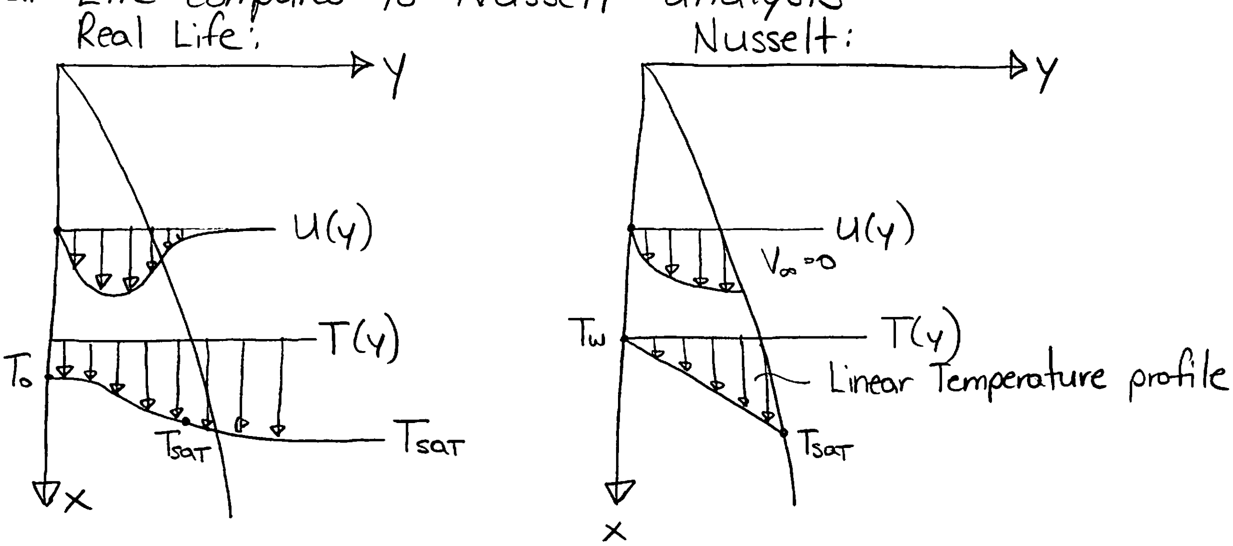


To account for the effect of Pr,

$$\boxed{h_{fg}' = h_{fg}\left[1 + (0.68 - 0.228/Pr)\, Ja\right]} \quad \text{for } Pr \geq 0.6$$

END OF LECTURE 20

5) *Real Life compared to Nusselt analysis*



6) *The presence of non-condensable gases acts to severely degrade heat transfer. When the condensig vapor is mixed with noncondensable air, uncondensed air must constantly diffuse away from the condensing film and vapor must diffuse inward toward the film. This coupled diffusion process*