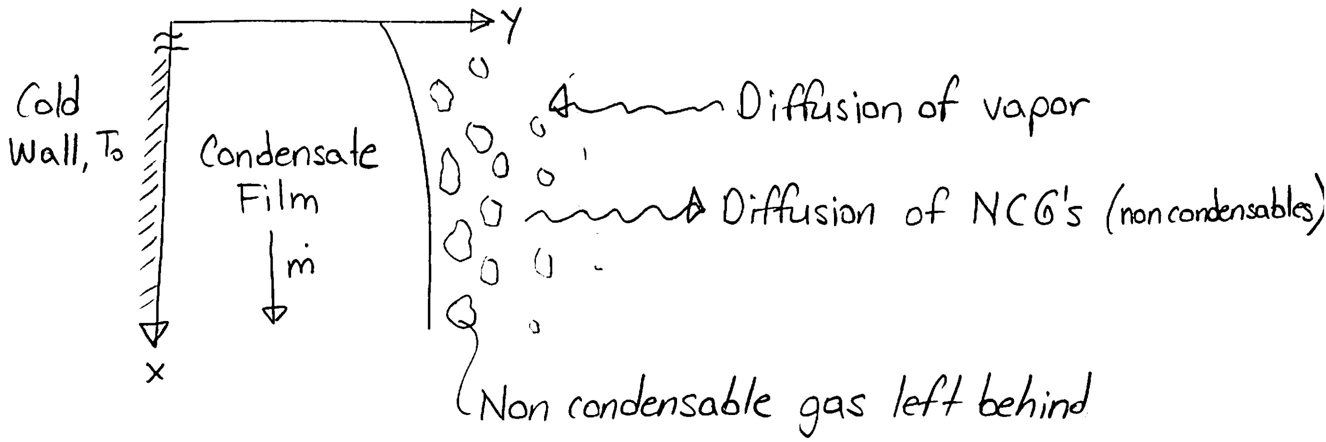
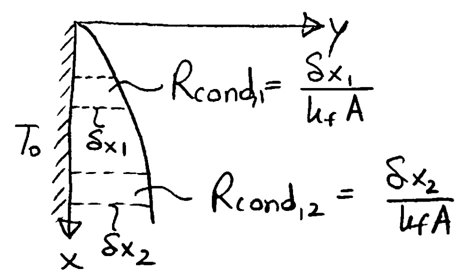
can considerably slow condensation. The resulting h can be cut by a factor of 5 if there is as little as 5% by mass of air mixed into the steam.



7) We see that $h_x = \frac{k_f}{\delta} \sim \frac{1}{x^{1/4}}$

So $h\downarrow$ as $x\uparrow$. This is just like laminar boundary layer theory and indicates that thicker condensate films add conduction resistance.



$R_{cond,1} = \frac{\delta_{x_1}}{k_f A}$

$R_{cond,2} = \frac{\delta_{x_2}}{k_f A} \Rightarrow R_{cond,2} > R_{cond,1}$ since $\delta_{x2} > \delta_{x1}$

So having shorter surfaces and many of them is benefitial.

Example | Water at 1 atm condenses on a strip 30cm high that is held at 90°C. Calculate Q/meter, δ (x = 30cm), and $\dot{m}$/meter.

$$\delta = \left[\frac{4k(T_{SAT} - T_o)\mu_f x}{\rho_f(\rho_f - \rho_g) g h_{fg}'}\right]^{1/4}$$

$$h_{fg}' = 2257\left(1 + \left(0.68 - \frac{0.228}{1.86}\right)\frac{4.211(10)}{2257}\right) = 2281 \text{ kJ/kg}$$

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So: $\delta = \left[\dfrac{4(0.677)(10)(2.99\times10^{-4})x}{961.9(961.9-0.6)(9.81)(2281\times10^3)}\right]^{1/4} = 0.000141x^{1/4}$

$\boxed{\delta(L) = 0.104\text{mm} = 104\mu m}$ ⇒ Thickness of your hair is 100μm!

$\overline{Nu}_L = \dfrac{4}{3}\dfrac{L}{\delta} = \dfrac{4(0.3m)}{3(0.000104m)} = 3846$

$q'' = \dfrac{\overline{Nu}_L k\Delta T}{L} = 86.8\,kW/m^2$ or $\boxed{\bar{h}_L = 8.68\,kW/m^2\cdot K}$

$Q_{TOT} = (86.8kW/m^2)(0.3m) = \boxed{26.0\ kW/m}$

For condensate flow rate per meter:

$$\boxed{\dot{m} = \frac{Q}{h_{fg}} = \frac{26.0}{2281} = 0.0114\,kg/m\cdot s}$$

Note how high $\bar{h}_L$ is for condensation phase change:

$$\boxed{\bar{h}_L = 8.68\,kW/m^2\cdot K \gg \bar{h}_{L,\text{convection}}}$$

Typically, phase change heat transfer processes are much higher in $\bar{h}$ due to the enormous amount of energy of latent heats of phase change.

Turbulent Film Condensation

We can define a Reynolds number for condensation as:

$Re_\delta = \dfrac{\rho_f V_f D_h}{\mu_f} = \dfrac{V_f D_h}{\nu_f} = \dfrac{Q D_h}{\nu_f A}$, Where Q = volumetric flow rate, A = area cross section

$D_h = \dfrac{4A}{P}$, $P = w$ (wetted perimeter, only the wall width, w)

$Q = \dfrac{\dot{m}}{\rho_f} \Rightarrow Re_\delta = \dfrac{\dot{m}}{\cancel{\rho_f}}\cdot\dfrac{\cancel{\rho_f}}{\mu_f}\cdot\dfrac{4\cancel{A}}{P\cancel{A}} \Rightarrow \boxed{Re_\delta = \dfrac{4\dot{m}}{\mu_f w}}$ w = width of condensing plate.

Now we can solve for Re_δ explicitly since we know $\dot{m}$.

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$$\dot{m} = \frac{w \rho_f(\rho_f-\rho_g) g \delta^3}{3\mu_f}$$

$$\boxed{Re_\delta = \frac{4 g \delta^3}{3 \nu_f^3}} \Rightarrow$$

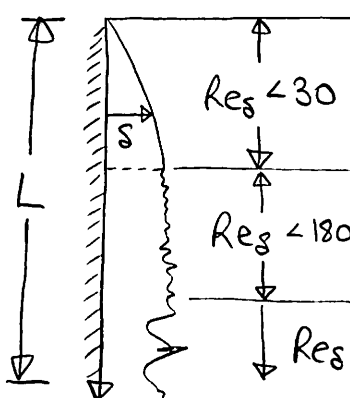
$$\boxed{L_c = \text{condensation length scale} = \left(\frac{\nu_f^2}{g}\right)^{1/3}}$$

Experimental correlations for Re_δ:

$$Re_\delta = 3.78\left[\frac{k_f L \Delta T}{\mu_f h'_{fg} (\nu_f^2/g)^{1/3}}\right]^{3/4} ; \quad Re_\delta < 30$$

$$Re_\delta = \left[\frac{3.7 k_f L \Delta T}{\mu_f h'_{fg} (\nu_f^2/g)^{1/3}} + 4.8\right]^{0.82} ; \quad 30 \leq Re_\delta \leq 1800$$

$$Re_\delta = \left[\frac{0.069 k_f L \Delta T}{\mu_f h'_{fg} (\nu_f^2/g)^{1/3}} \cdot Pr_f^{0.5} - 151 Pr_f^{0.5} + 253\right]^{4/3} ; \quad Re_\delta > 1800$$



$Re_\delta < 30$ (Laminar) ; $\overline{Nu} = \left(\frac{3}{4} Re_\delta\right)^{-1/3}$

$Re_\delta < 1800$ (Transition) ; $\overline{Nu} = 0.822\, Re_\delta^{-0.22}$

$Re_\delta > 1800$ (Turbulent) ; $\overline{Nu} = 3.8 \times 10^{-3} \cdot Re_\delta^{0.4} Pr_f^{0.65}$

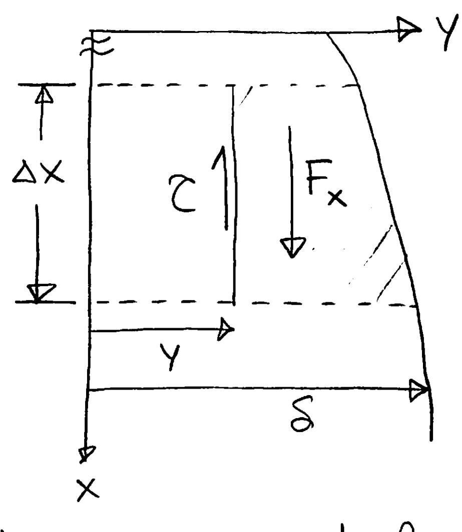
Where $\boxed{\overline{Nu} = \frac{\bar{h}_L L_c}{k_f}} ; \boxed{L_c = \left(\frac{\nu^2}{g}\right)^{1/3}}$

*May need itteration to solve.

Film Condensation on Arbitrary Shapes

We can generalize our previous approach to any body undergoing condensation.

Looking at our condensate more closely



F_x = body force per unit volume
For exampe, a vertical flat plate would have: $F_x = \underbrace{g}_{\text{gravity}}\underbrace{(\rho_f - \rho_g)}_{\text{Boyancy}}$

Applying a general force balance on the shaded area, we obtain:

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$$\tau = \mu \frac{\partial u}{\partial y} \Delta x$$

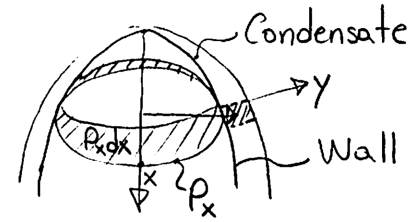
$$\mu \frac{\partial u}{\partial y} \cancel{\Delta x} = F_x (\delta - y) \cancel{\Delta x}$$

$$\mu \, \partial u = F_x (\delta - y) \, dy$$

$$u = \frac{F_x}{\mu} \left(\delta y - \frac{y^2}{2} \right)$$

The mass flow rate is: (P_x = perimeter of body at location x or wetted perimeter)

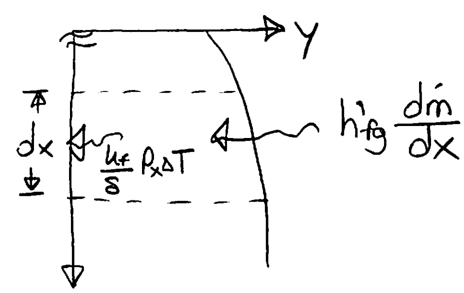
$$\dot{m} = \rho_f \int_0^\delta u P_x \, dy = P_x F_x \left(\frac{\rho_f}{3\mu} \right) \delta^3 \quad ①$$



Note, $P_x dy$ = Area cross section

Again using the energy equation we developed previously

$$\underbrace{\frac{k_f}{\delta} P_x \Delta T}_{\text{Conduction at the wall}} = \underbrace{h_{fg}' \frac{d\dot{m}}{dx}}_{\text{Latent heat}} \quad ②$$



Now we can re-arrange eq. ①

$$\delta = \dot{m}^{1/3} \left[\frac{3\mu_f}{\rho_f P_x F_x} \right]^{1/3} \quad ③$$

Substituting ③ into ② and integrating yields the total condensate flow rate:

$$\dot{m} = \frac{4^{3/4}}{3} \left[\frac{\rho_f k_f^3 \Delta T^3}{\mu_f h_{fg}'^3} \right]^{1/4} \left[\int_0^L P_x^{4/3} F_x^{1/3} \, dx \right]^{3/4}$$

Finally, since:

$$\dot{m} h_{fg}' = \bar{h} \left(\int_0^L P_x \, dx \right) \Delta T = \bar{h} A_{cond} \Delta T$$

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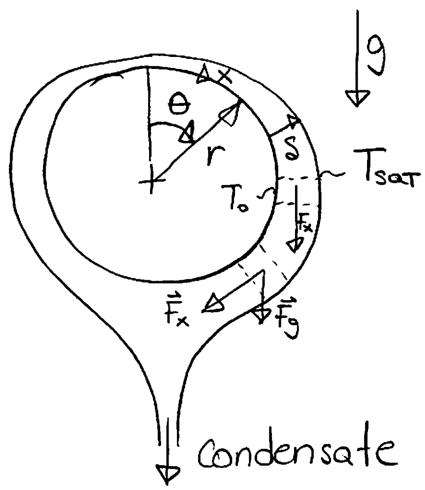
We can re-write the average heat transfer coefficient as:

$$\boxed{\bar{h} = \frac{4^{3/4}}{3} \cdot \frac{1}{A_{cond}} \left[\frac{\rho_f k_f^3 h_{fg}'}{\mu_f \Delta T}\right]^{1/4} \left[\int_0^L P_x^{4/3} F_x^{1/3}\, dx\right]^{3/4}}$$

↳ Condensation $\bar{h}$ for any arbitrary body undergoing filmwise laminar cond.

$$\boxed{A_{cond} = \int_0^L P_x\, dx}$$ ⇒ Condensation Area

Example | Find $\bar{h}$ for a horizontal tube.



$dx = r\,d\theta$

$F_x = g(\rho_f - \rho_g)\sin\theta$

$P_x = 1$

$A_{cond} = \int_0^{\pi} 1\, r\, d\theta = \pi r$

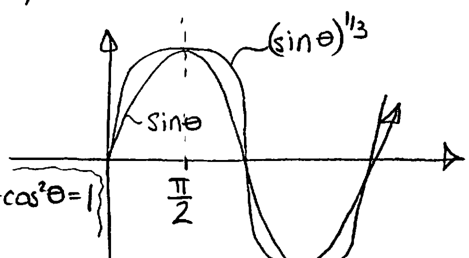
↳ Note didn't integrate to 2π due to symmetry.

Now we can solve:

$$\bar{h} = \frac{4^{3/4}}{3} \cdot \frac{1}{\pi r} \left[\frac{\rho_f k_f^3 h_{fg}'}{\mu_f \Delta T}\right]^{1/4} \left[\int_0^{\pi} \sin^{1/3}\theta\, d\theta\right]^{3/4} \cdot \underbrace{r^{3/4}}_{\text{From } dx = r d\theta} \left[g(\rho_f - \rho_g)\right]^{1/4}$$

The integral here is not trivial to solve, so we can use a trick:

$$\int_0^{\pi} (\sin\theta)^{1/3}\, d\theta = 2\int_0^{\pi/2} (\sin\theta)^{1/3}\, d\theta$$



Let $s = \sin^2\theta$, $ds = 2\sin\theta\cos\theta\, d\theta$, $\sin^2\theta + \cos^2\theta = 1$

$$2\int_0^{\pi/2} (\sin\theta)^{1/3} d\theta = \int_0^1 s^{\frac{2}{3}-1} (1-s)^{\frac{1}{2}-1}\, ds$$ ⇒ Looking up in integral table or see page 129 of our notes.

$$= \frac{\Gamma(2/3)\,\Gamma(1/2)}{\Gamma(2/3 + 1/2)} = 2.5871, \quad \Gamma = \text{Gamma function}$$

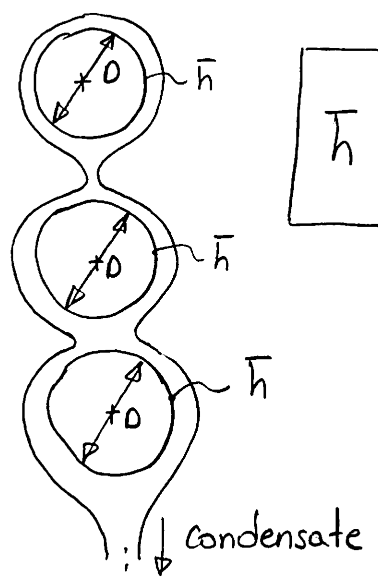
Now we can solve for $\bar{h}$

$$\bar{h} = \frac{4^{3/4}}{3}\,\frac{2^{1/4}}{\pi}\,(2.5871)^{3/4}\left[\frac{\rho_f(\rho_f-\rho_g)g k_f^3 h_{fg}'}{D\mu_f \Delta T}\right]^{1/4}, \quad D = \text{cylinder diameter}$$

$$\boxed{\bar{h} = 0.728\left[\frac{\rho_f(\rho_f-\rho_g)g h_{fg}' k_f^3}{D\mu_f \Delta T}\right]^{1/4}}$$

$\Rightarrow$ heat transfer coefficient for laminar film condensation on the outside of horizontal cylinder.

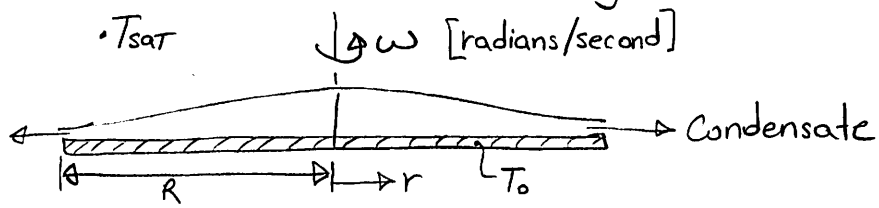
Note, if we had a tube bundle, with a continuous liquid condensate film and n tubes



$$\boxed{\bar{h} = 0.728\left[\frac{\rho_f(\rho_f-\rho_g)g h_{fg}' k_f^3}{nD\mu_f \Delta T}\right]^{1/4}}$$

$\hookrightarrow$ n = number of tubes in series.

Example | Condensation on a rotating disc in zero gravity



For this case; $A_{cond} = \pi R^2$ (Total condensation area)

$P_x = 2\pi r$ (Perimeter of wetted area at location x)

$F_x = \rho_f r\omega^2$ (Body force per unit volume)

$dx = dr$

We can first evaluate our integral in our generalized equation for $\bar{h}$:

$$\int_0^L P_x^{4/3} F_x^{1/3}\,dx = (2\pi)^{4/3}(\rho_f \omega^2)^{1/3}\int_0^R r^{4/3} r^{1/3}\,dr$$

$$= (2\pi)^{4/3}(\rho_f\omega^2)^{1/3}(3/8)R^{8/3}$$

$$\frac{1}{A_{cond}}\cdot\left[\int_0^L P_x^{4/3}F_x^{1/3}\,dx\right]^{3/4} = \frac{2\cancel{\pi}}{\cancel{\pi}R^2}\left(\frac{3}{8}\right)^{3/4}R^2(\rho_f\omega^2)^{1/4} = 2\left(\frac{3}{8}\right)^{3/4}(\rho_f\omega^2)^{1/4}$$

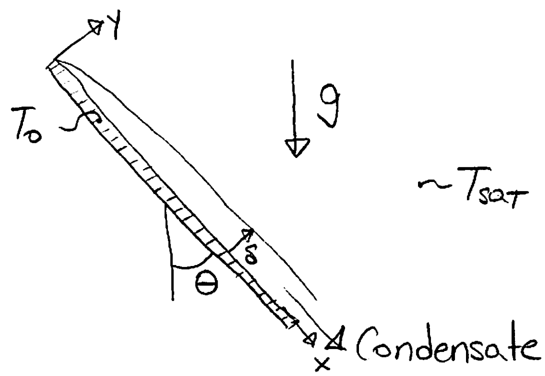
Back substituting, we obtain:

$$\boxed{\bar{h} = \left[\frac{2k_f^{\,3}(\rho_f\omega)^2 h'_{fg}}{3\mu_f\Delta T}\right]^{1/4}}$$

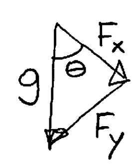
$\Rightarrow$ Average heat transfer coefficient over the entire surface.

END OF LECTURE 21

Example Inclined surface at angle θ from the vertical



$A_{COND} = (1)L = L$
$P_x = 1$
$F_x = (\rho_f - \rho_g)g\cos\theta$



Solving our integral term:

$$\int_0^L P_x^{4/3}F_x^{1/3}\,dx = [(\rho_f-\rho_g)g\cos\theta]^{1/3}\int_0^L dx = ((\rho_f-\rho_g)g\cos\theta)^{1/3}L$$

$$\bar{h} = \frac{4^{3/4}}{3}\frac{1}{L}\left[\frac{\rho_f k_f^{\,3}h'_{fg}}{\mu_f\Delta T}\right]^{1/4}\left[((\rho_f-\rho_g)g\cos\theta)^{1/3}L\right]^{3/4}$$