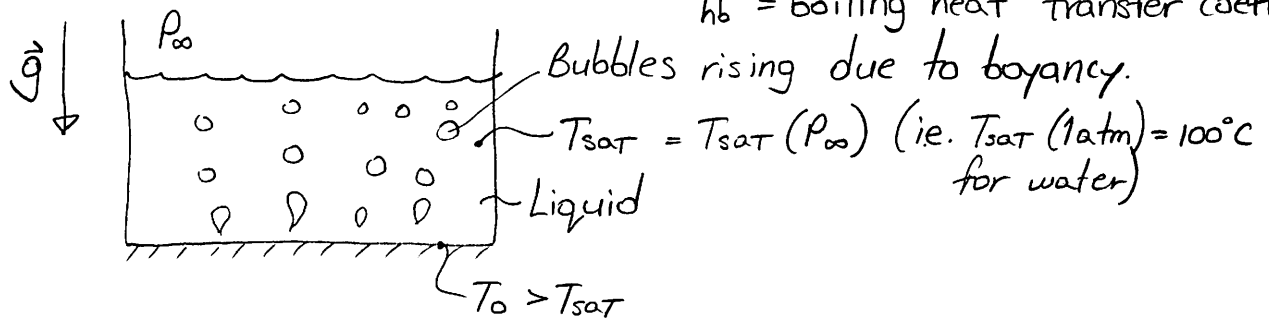Nenad Miljkovic

Boiling Heat Transfer

Very important in power generation systems (Rankine cycle). Opposite of condensation since heat is added to a liquid which changes phase to a vapor. More common than condensation due to it's daily use in the kitchen for cooking.

In terms of heat transfer, we characterize boiling as:

$$q''_b = h_b (T_0 - T_{sat}) \; ; \quad T_0 = \text{surface temperature}$$

T_{sat} = fluid saturation temperature at that pressure.

h_b = boiling heat transfer coeff.

Diagramatically:

Bubbles rising due to boyancy.

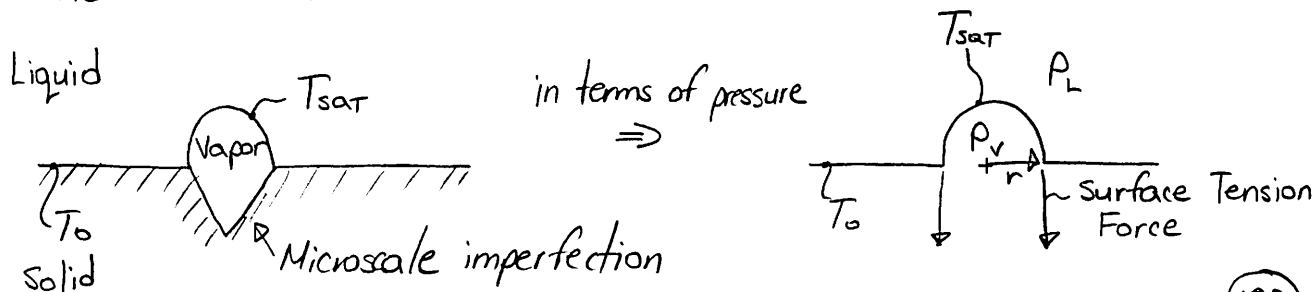$T_{sat} = T_{sat}(P_\infty)$ (ie. $T_{sat}(1 atm) = 100°C$ for water)

$T_0 > T_{sat}$

In general, boiling is characterized by the vapor bubbles that grow and detach from the surface. This depends on:

1) The temperature difference, $T_0 - T_{sat}$
2) The nature of the surface (i.e. rough, smooth, pitted, etc...)
3) Thermophysical properties of the fluid (i.e. surface tension).

Initiation of Boiling, Bubble Growth

Boiling initiates by the formation of vapor bubbles at imperfections at the surface.

Microscale imperfection

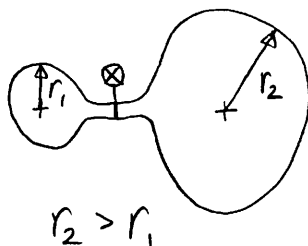in terms of pressure $\Rightarrow$

Surface Tension Force

If we apply a force balance on our bubble:

$$\underbrace{(P_v - P_L)\pi r^2}_{\text{Pressure Force}} = \underbrace{2\pi r \sigma}_{\text{Surface Tension Force}} \quad ; \quad \sigma = \text{fluid surface tension } [N/m]$$
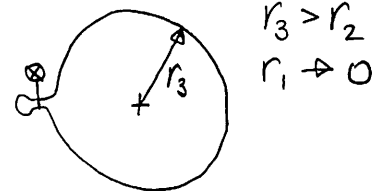
$$\boxed{\Delta P = \frac{2\sigma}{r}} \Rightarrow \text{Pressure difference across the liquid-vapor interface.}$$

Note, as $r \to 0$, $\Delta P \to \infty$, meaning that small bubbles have a much larger internal pressure.

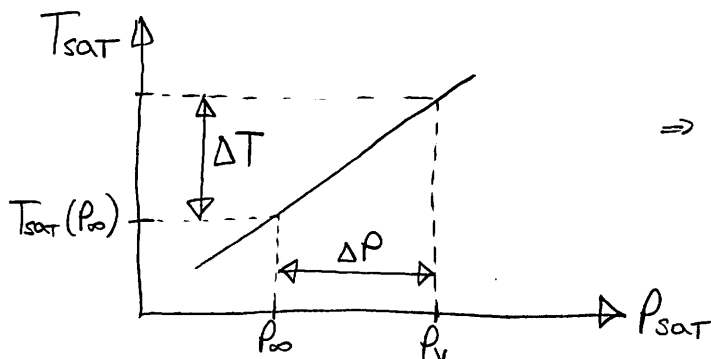Aside: If I connect 2 baloons with r_1 & r_2 together, & $r_1 < r_2$, what will happen:

$r_2 > r_1$

Answer
$\Rightarrow$

$r_3 > r_2$
$r_1 \to 0$

Since $P_{v,1} > P_{v,2}$, the small baloon will drain into the big one.

The ΔP we've calculated is needed to create a vapor bubble of that given size, and overcome the bottleneck. So how can we relate this to boiling and surface temperature (T_o)?

Fluid Saturation curve:

$\Rightarrow$ The ΔT you need to create a bubble nucleation site corresponding to ΔP.

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So as you increase your ΔT, you create more nucleation sites with smaller radii and increase bubble creation frequency from existing sites.

We can relate our ΔP to ΔT through the Clausius-Clapeyron relation:

$$\boxed{\left(\frac{dT}{dP}\right)_{SAT} = \frac{\Delta \upsilon \, T_{sat}}{h_{fg}} \approx \frac{T_{sat}}{\rho_v \, h_{fg}}}$$

$\Delta \upsilon$ = specific volume change
(since $\Delta \upsilon = \upsilon_g - \upsilon_f \approx \upsilon_g$ since $\upsilon_f \ll \upsilon_g$)

Aside: Derivation of the Clausius-Clapeyron relation
From thermodynamics of a state in equilibrium (state postulate)

$$ds = \left(\frac{\partial s}{\partial \upsilon}\right)_T d\upsilon + \left(\frac{\partial s}{\partial T}\right)_\upsilon dT \quad , \quad \begin{array}{l} s = \text{specific entropy} \\ \upsilon = \text{specific volume} \\ T = \text{temperature} \end{array}$$

For phase change, $dT = 0$

$$ds = \left(\frac{\partial s}{\partial \upsilon}\right)_T d\upsilon$$
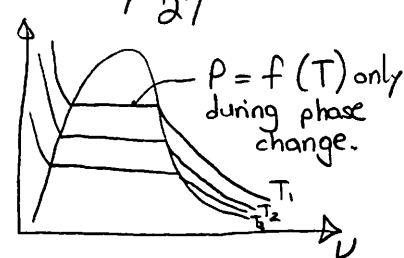
Using the appropritate Maxwell relation: $+\left(\frac{\partial s}{\partial V}\right)_T = +\left(\frac{\partial P}{\partial T}\right)_\upsilon$

$$ds = \left(\frac{\partial P}{\partial T}\right)_\upsilon d\upsilon$$

Since P, T = constant during a phase transition, $\frac{\partial P}{\partial T}$ = constant for different υ. So now we can say:

$$\int ds = \int \left(\frac{dP}{dT}\right)_{SAT} d\upsilon$$

$$\Delta S = \left(\frac{dP}{dT}\right)_{SAT} \Delta \upsilon \quad , \quad \Delta S = S_g - S_f, \; \Delta \upsilon = \upsilon_g - \upsilon_f$$

P = f(T) only during phase change.

For a closed system undergoing an internally reversible process:

$$du = \delta q - \delta w = T ds - P d\upsilon \quad , \quad u = \text{specific energy}$$

We know specific enthalpy, h, as: $dh = du + Pdu$

$$dh = Tds \Rightarrow \int_f^g ds = \int_f^g \frac{dh}{T} \Rightarrow \Delta S = \frac{\Delta h}{T}, \quad \begin{array}{l} \Delta S = S_g - S_f \\ \Delta h = h_g - h_f \end{array}$$

For a phase change process: $h_g - h_f = h_{fg}$

$$\Delta S = \frac{h_{fg}}{T_{sat}} = \left(\frac{dT}{dP}\right)_{sat} \Delta U$$

So: $$\boxed{\left(\frac{dT}{dP}\right)_{sat} = \frac{\Delta U \, T_{sat}}{h_{fg}}} \Rightarrow \text{Clausius - Clapeyron relation.}$$

So back to our boiling derivation
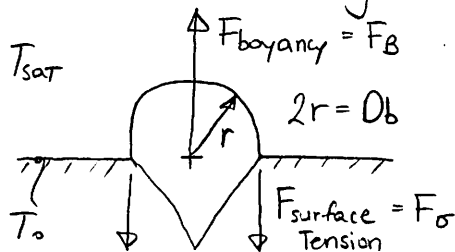
$$\Delta T = \frac{\Delta P \, T_{sat}}{p_v h_{fg}}$$

We know for a bubble that $\Delta P = \frac{2\sigma}{r}$, so

$$\boxed{\Delta T = \frac{2\sigma T_{sat}}{r p_v h_{fg}}}, \quad \Delta T = T_0 - T_{sat}$$
$\Rightarrow$ Temperature difference required to nucleate a bubble of radius r.

Note, often ΔT is referred to as "wall superheat."

Bubble Departure Dynamics

For a bubble sitting on a surface, we can do a force balance
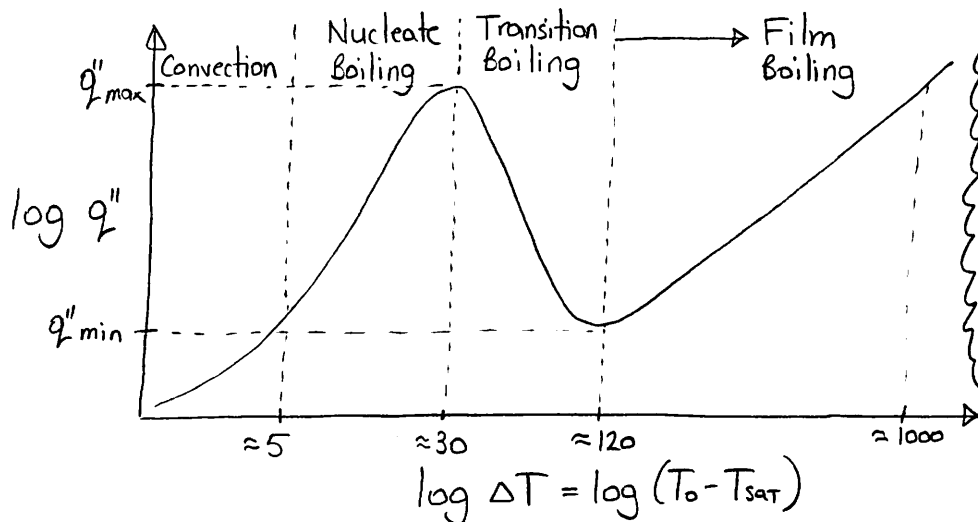

$F_B \sim D_b^3 (p_f - p_g)g$

$F_\sigma \sim \Delta P A_{cs} \sim \frac{2\sigma}{D_b/2} \cdot \frac{D_b^2}{4} \sim D_b \sigma$

$F_B = F_\sigma \Rightarrow \boxed{D_b \sim \left[\frac{\sigma}{g(p_f - p_g)}\right]^{\frac{1}{2}}}$

D_b = bubble departure diameter in boiling heat transfer.

186

<u>Heat Transfer</u>: Now we can say something about the heat transfer taking place from the wall to the fluid. Experimentally, researchers have discovered different regiemes of boiling: (note, values of ΔT shown are for water.)

Aside:
Note, in the convection regeime, boiling does not occur since ΔT is too low to create bubbles and natural convection removes the heat from the hot plate.

We will discuss each regieme in detail.

<u>Nucleate Boiling:</u>
For a hemispherical bubble $(\theta = 90°)$, a force balance reveals:

$$2\pi\sigma R_b = \frac{2}{3}\pi R_b^3 (\rho_f - \rho_g)g \quad ; \quad R_b = \frac{D_b}{2}$$

$$R_b = \left[\frac{3\sigma}{(\rho_f - \rho_g)g}\right]^{1/2} \Rightarrow \text{Bubble departure size}$$
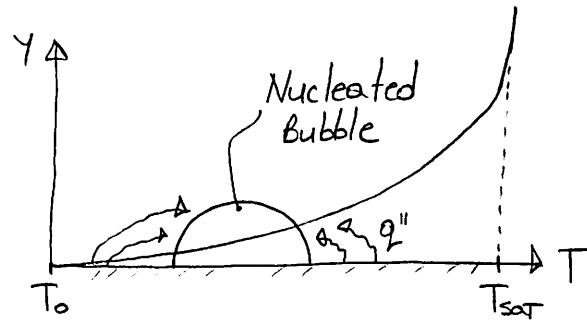
For our analysis, the characteristic length for boiling will be taken as:

$$\boxed{L_c = \left[\frac{\sigma}{(\rho_f - \rho_g)g}\right]^{1/2}} \Rightarrow \text{Boiling characteristic length}$$

$$Nu = \frac{hL_c}{k_f} \Rightarrow \text{Boiling Nusselt number}$$

So the rate of bubble growth depends on the convective heat transfer through the liquid to the liquid-vapor interface

required to supply the enthalpy of vaporization. Physically, you can think of it as:

Since convection is involved, we expect the fluid Prandtl number to be a relavant parameter.

In addition, similar to condensation, we end up sensibly heating the fluid and not all energy goes into phase change, hence the Jacob number is a relevant parameter.

$$Ja_f = \frac{c_{p,f}(T_o - T_{sat})}{h_{fg}}$$

Experiments by Rohsenow (the father of boiling) show:

$$\boxed{Nu = \frac{Ja_f^2}{C_{nb}^3 \, Pr_f^m} = \frac{h \, L_c}{k_f}}$$

$\Rightarrow$ C_{nb} and m are liquid and surface properties (Table 7.2, page 692 in Mills).

*All properties evaluated at T_{sat}

For water, $m \approx 2.0$

$C_{nb} \approx 0.007$ to 0.013 (depends on surface material).

We see that: $q'' = h\Delta T \sim \Delta T^3$ for boiling. This strong dependence on ΔT is due to the rapid activation of nucleation sites at elevated ΔT (since dP/dT is very large)

Example | Determine h and q'' for water boiling on a polished copper pot at 390 K and 1 atm.

$$\Delta T = T_o - T_{sat} = 390K - 373.15K = 16.85 \ K$$

We see from our graph on page 187 of notes that we are in the nucleate boiling regieme $(5 < \Delta T < 30)$. Using our Rohsenow correlation:

$$Nu = \frac{Ja^2}{C_{nb}^3 \ Pr^m}$$

$\Rightarrow$ All properties are evaluated at T_{sat}

$T_{sat} = 373.15 \ K$
$\sigma = 58.9 \times 10^{-3} \ N/m$
$h_{fg} = 2.257 \times 10^6 \ J/kg$
$k_f = 0.681 \ W/m \cdot K$
$\rho_f = 958 \ kg/m^3$
$C_{p,f} = 4212 \ J/kg \cdot K$
$Pr_{,f} = 1.76$

$$Ja = \frac{C_{p,f}(T_o - T_{sat})}{h_{fg}} = \frac{(4212)(390-373.15)}{(2.257 \times 10^6)} = 3.145 \times 10^{-2}$$

From table 7.2 in Mills, $C_{nb} = 0.013, \quad m = 2.0$

$$Nu = \frac{(3.145 \times 10^{-2})^2}{(0.013)^3 (1.76)^2} = 145$$

$$L_c = \left[\frac{\sigma}{(\rho_f - \rho_g)g}\right]^{1/2} = 2.5 \times 10^{-3} m$$

$$h = \frac{k_f}{L_c} Nu = \left(\frac{0.681}{2.5 \times 10^{-3}}\right)(145) = 39\,500 \ W/m^2 \cdot K \ \text{or} \ \boxed{39.5 \ kW/m^2 \cdot K}$$

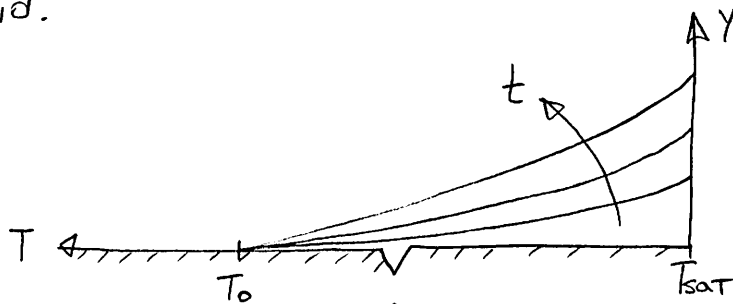$$q'' = h\Delta T = 6.66 \times 10^5 \ W/m^2 = \boxed{666 \ kW/m^2 = q''}$$

Note, q'' and h are extremely high values. On the same order as condensation values. Much higher than h_{conv}, h_{rad}.

Looking at a more fundamental boiling model:

Mikic – Rohsenow Model (for nucleate boiling)

Bora Mikic (the professor who taught me this class) developed this model as part of a heat transfer final exam. Warren Rohsenow didn't know how to solve it so he asked and when Mikic explained, Rohsenow saw the brilliance of it and published it together.

The model is based on transient heat conduction into the fluid.

⟹ Mikic assumed that a majority of the heat is transfered to the thin fluid next to the surface.

We solved in the first part of the course that: (pg. 66 of notes)

$$q'' = \frac{h_f \Delta T}{\sqrt{\pi \alpha_f t}} \Rightarrow \text{heat flux due to transient heat transfer}$$

If we assume a frequency of bubble formation as f

$$f = \frac{1}{C} \quad , \quad \alpha_f = \frac{h_f}{\rho_f C_{p,f}}$$

$$q'' = \frac{h_f \Delta T}{\sqrt{\pi \frac{h_f}{\rho_f C_{p,f}} t}} \Rightarrow \text{Note, this is instantaneous heat flux at } t.$$
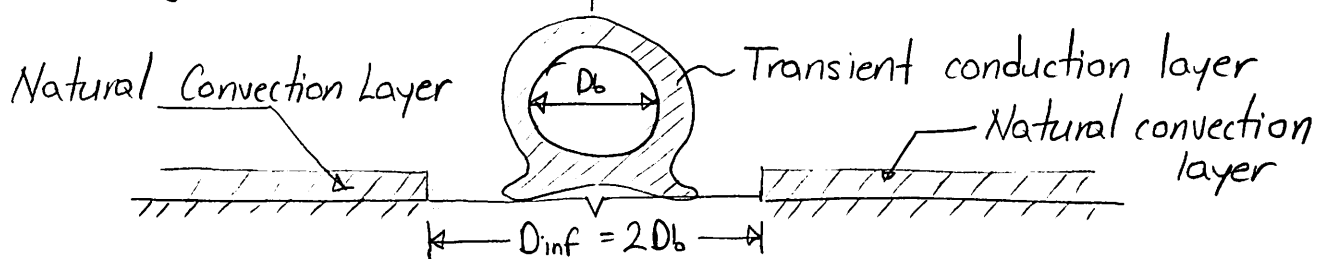
$$q''_{AVE} = \frac{1}{C} \int_0^{C} q'' \, dt = f \int_0^{\frac{1}{f}} q'' \, dt \Rightarrow \text{Average heat flux for one nucleation site.}$$

$$q''_{AVE} = 2 \frac{h_f \Delta T}{\sqrt{\pi \frac{h_f}{\rho_f C_{p,f}}}} \cdot \sqrt{f}$$

So in the Mikic model, the bubbles departing act to bring in fresh cool (T_{SAT}) water to the surface, and re-start the transient conduction process.

Assuming that the influence area of the bubble is πD_b^2

The bubble rising drags away πD_b^2 area of the superheated layer. So now we can solve for overall area heat flux:

$$q''_{TOT} = \pi D_b^2 \, n \, q''_{AVE} \quad ; \quad n = \text{number of active sites per } m^2$$

$$\boxed{q''_{TOT} = 2\sqrt{\pi} \sqrt{k_f \rho_f C_{p,f}} \sqrt{f} \; D_b^2 \cdot n \cdot \Delta T} \implies \text{Mikic-Rohsenow nucleate boiling model.}$$

The model is accurate when there is no overlap between transient conduction layers.

It's also more difficult than our previous Rohsenow correlation since f, and n need to be known a-priori.

Advantages of this model is that it is a lot more physical, and doesn't depend on tabulated parameters or fitting coefficients.

For more details, see Mikic & Rohsenow's paper on the dropbox.

———————— END OF LECTURE 23 ————————