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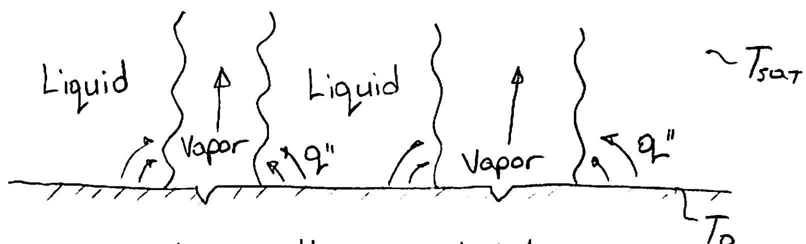
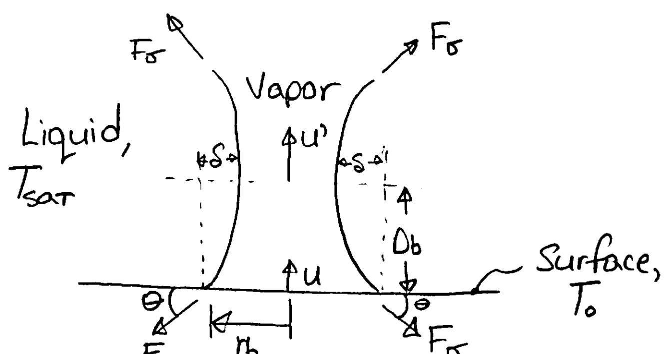
Critical Heat Flux (q''_{max})

We see on our boiling curve that the nucleate boiling regeime reaches a maximum heat flux (at $\Delta T \approx 30$ for water) and then decreases. The reason for this peak is due to a vapor instability in rising bubble columns on the surface.

At high heat fluxes, bubbles coalesce as they rise (since f is very high) and form isolated columns:



We can analyze the stability of these columns. Looking at one column:



<u>Assuming</u>: No vapor generation upstream of the column. All vapor is generated at column base at u, near the contact line.

By conservation of mass:

$$u \cancel{\pi} r_b^2 = u' \cancel{\pi} (r_b - \delta)^2$$

$$u' = u\left(\frac{r_b}{r_b-\delta}\right)^2 = u\left(\frac{1}{1-\delta/r_b}\right)^2 \quad \text{, Assuming } \frac{\delta}{r_b} \ll 1$$

By Maclaurin series expansion (Taylor series expansion about $x=0$): $\boxed{\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots}$

$$u' \cong u\left(1 + 2\frac{\delta}{r_b}\right),$$

Nenad Miljkovic

Using the Bernoulli equation along a streamline inside the vapor column from u to u'

$$\Delta P = \frac{1}{2}\rho_g\left(u'^2 - u^2\right) \quad (1)$$

$$u'^2 = u^2\left(1 + \frac{4\delta}{r_b} + 4\frac{\delta^2}{r_b^2}\right)$$

Since $\frac{\delta}{r_b} \ll 1$, $\left(\frac{\delta}{r_b}\right)^2 \approx 0$, so: $u'^2 = u^2\left(1 + \frac{4\delta}{r_b}\right)$ (2)

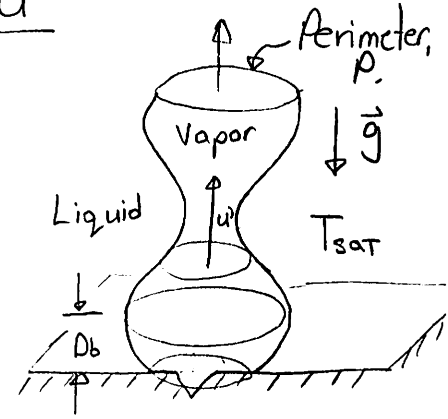
Back substituting (2) into (1), we obtain:

$$\Delta P = \frac{1}{2}\rho_g\left(\cancel{u^2} + \frac{4\delta u^2}{r_b} - \cancel{u^2}\right) = \frac{2\rho_g \delta u^2}{r_b}$$

Now we can calculate a force balance:

$$F_{\Delta p} = 2\rho_g u^2 \frac{\delta}{\cancel{r_b}}\left(\pi r_b^{\cancel{2}}\right) = 2\rho_g u^2 \delta \pi r_b$$

$$F_\sigma = P\sigma\sin\theta \approx P\sigma\tan\theta = \sigma\frac{\delta}{2\cancel{r_b}}\cdot 2\pi\cancel{r_b}$$



3-D Vapor Column

Note, here $F_{\Delta p}$ & F_σ are forces on the CV of the vapor column, and we assume that $\theta \to 0$, so $\sin\theta \approx \tan\theta$. (For high heat fluxes, $\theta \to 0$ and our column is nearly flat and vertical)

By force ballance: $F_{\Delta p} \sim F_\sigma$

$$2\cancel{\pi} r_b \rho_g u^2 \cancel{\delta} \sim \cancel{\pi}\sigma\cancel{\delta} \quad (D_b = 2r_b)$$

$$u^2 \sim \frac{\sigma}{\rho_g D_b} \sim \frac{\sqrt{\sigma g(\rho_f - \rho_g)}}{\rho_g} \quad \left(\text{since } D_b \sim \left[\frac{\sigma}{(\rho_f - \rho_g)g}\right]^{1/2}\right), \text{pg } 187$$

So $u \sim \frac{1}{\rho_g^{1/2}}\left[\sigma g(\rho_f - \rho_g)\right]^{1/4}$ ⇒ if velocity is $> u$, the column becomes unstable.

$$q''_{max} \sim u\rho_g h_{fg} \sim h_{fg}\rho_g^{1/2}\left[\sigma g(\rho_f - \rho_g)\right]^{1/4}$$

So in experiments, we look for the following form

$$\boxed{q''_{max} = C_{max} h_{fg} \left[\sigma g \rho_g^2 (\rho_f - \rho_g) \right]^{1/4}}$$

$\Rightarrow$ Critical heat flux based on Helmholtz instability.

This correlation works very well. For large flat surfaces, $C_{max} = 0.15$. In reality, it depends on the surface geometry and size. For more details, see pg. 694, 695 in Mills.

Note, this instability mechanism is also important in many other physical problems such as flags flapping. When the vapor columns become unstable, and the velocity induced pressure forces exceed the surface tension forces, the jet begins to oscilate (like a flag in the wind) and has the chance of merging with adjacent columns.

Example | Find q''_{max} for boiling water on a flat surface

$$q''_{max} = (0.15)(2.257\times10^6)\left[(0.0589)(9.81)(0.59)^2(958-0.59)\right]^{1/4}$$

$$\boxed{q''_{max} = 1.26\ MW/m^2 = 126\ W/cm^2}$$

↳ This is usually labeled as the critical heat flux, or q''_{CHF}.

Note, much research has been going on in the past 10 years to show that q''_{max} also depends on surface properties, such as wettability, surface roughness and oxidation.

Film Boiling

To understand the regieme of film boiling, we need to look at the history of the boiling curve.

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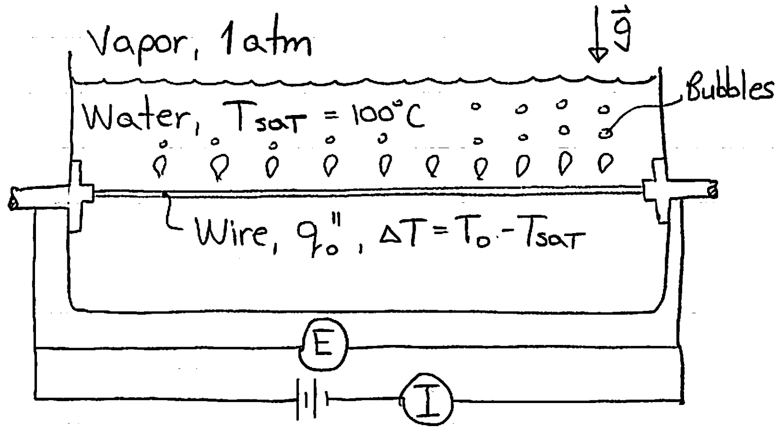
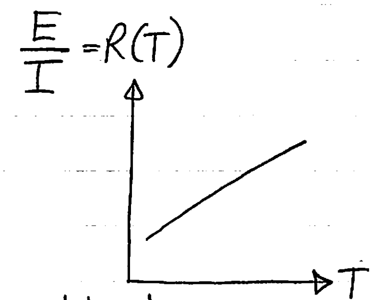
In 1934, Nukiyama did an experiment of boiling on a nichrome wire in a saturated pool of water:

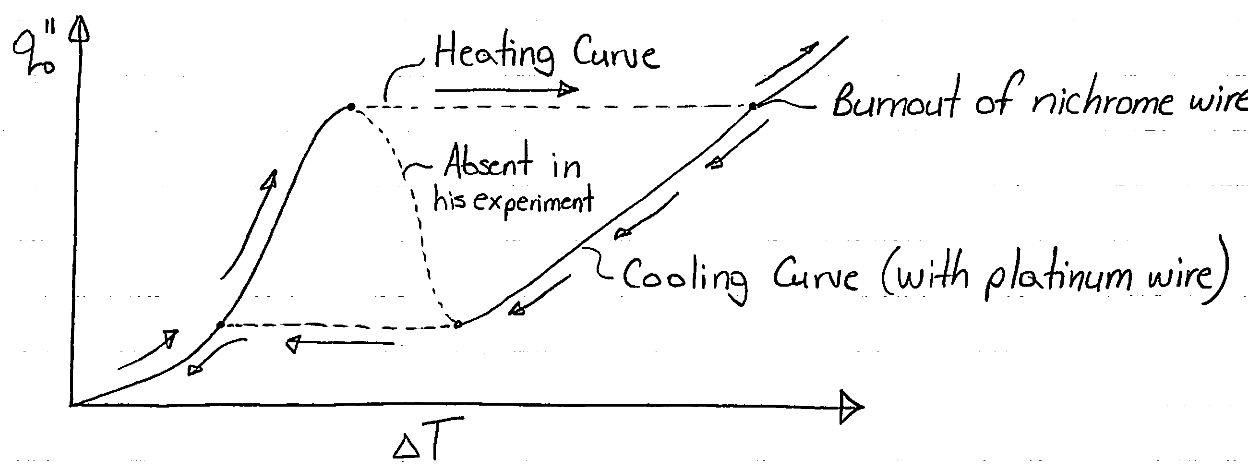




Calibration curve to determine T_o from E and I.

From this setup, ΔT was the dependent variable, and q_o'' was the independent or controled variable.

Nukiyama observed a fascinating phenomenon:



He noticed that at q''_{max}, the temperature of the wire T_o jumped drastically to $>1000°C$ and his wire melted.

He re-did the experiment with a platinum wire ($T_{melt} > 2000°C$) and once he cooled back down, he had a large hysteresis.

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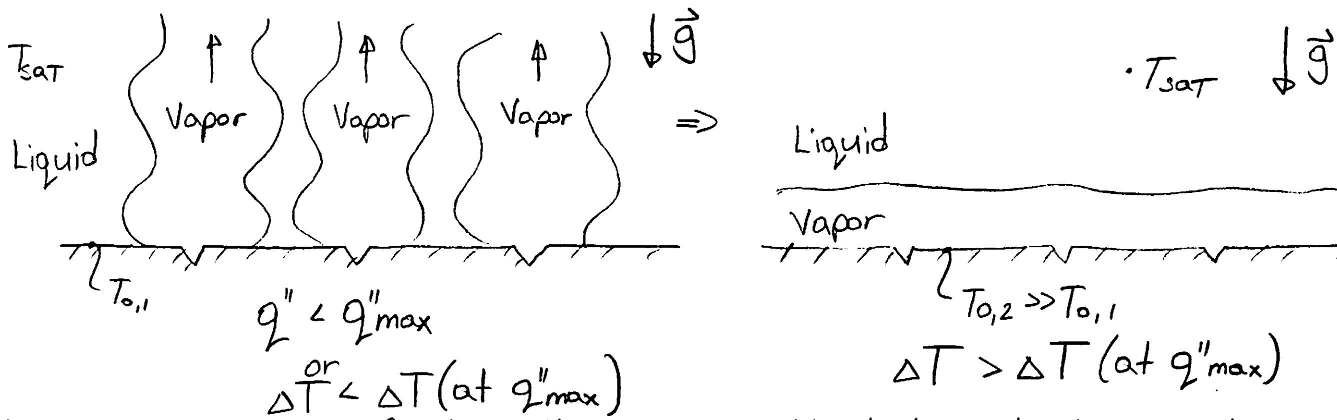
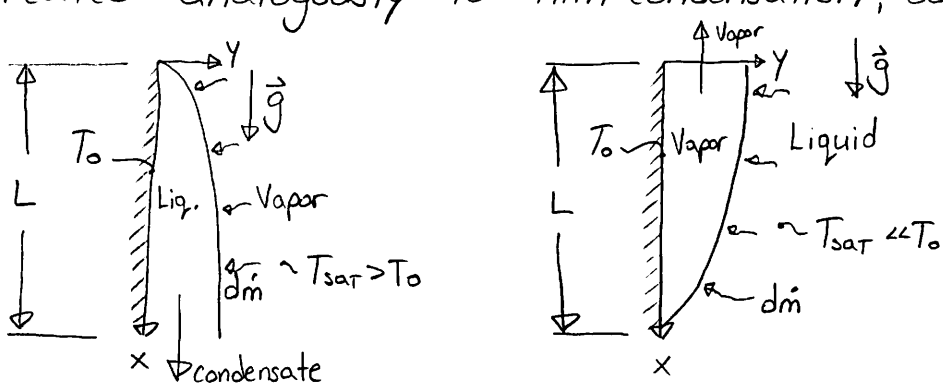
So what happened? The phenomena is related to the q''_{max} limit we discussed earlier. If we exceed q''_{max}, we have a Helmholtz instability which causes the merging of vapor columns and the formation of a thin vapor blanket.



The presence of the thin vapor blanket acts to severely impede heat transfer. Now the main mode is convection to vapor and radiation from the surface to the fluid. Since:

h_{conv} and $h_{rad} \ll h_{boiling}$, T_o jumps dramatically as Nukiyama observed.

The formation and growth of this thin vapor layer can be treated analogously to film condensation, but upside down.



We can show that:

$$\boxed{\bar{h} = C_{fb}\left[\frac{(\rho_f - \rho_g) g h'_{fg} k_g^3}{\nu_g L (T_o - T_{sat})}\right]^{1/4}}$$

$\Rightarrow$ Film boiling heat transfer coefficient

$$\boxed{h'_{fg} = h_{fg} + 0.35\, c_{p,v}(T_o - T_{sat})}$$

For a plane surface, $C_{fb} = 0.71$

For a cylinder, $L = D$, $C_{fb} = 0.62$

For a sphere, $L = D$, $C_{fb} = 0.67$

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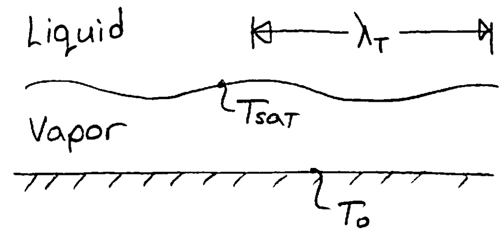
Minimum Heat Flux (q''_{min})

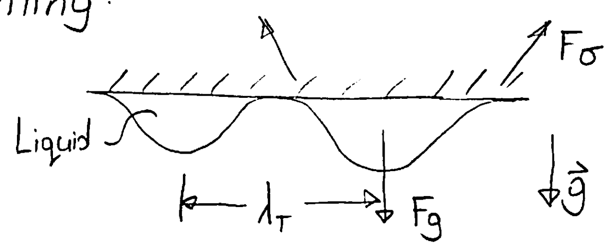
If we are in the film boiling regieme, and q'' or ΔT is reduced, at a certain point the film will break down. This occurs when the vapor generation rate becomes too low to sustain Taylor instability waves at the liquid-vapor interface.



The interface is unstable to wavelengths on the order of:

$$\lambda_T \sim L_c \sim \left[\frac{\sigma}{(\rho_f-\rho_g)g}\right]^{1/2}$$

This criterion is intuitive since its nothing but a force balance between gravity and surface tension; think of droplets on a ceiling:



$$F_\sigma \sim \sigma \pi \lambda_T$$
$$F_g \sim (\rho_f-\rho_g) g \tfrac{4}{3}\pi \lambda_T^3$$

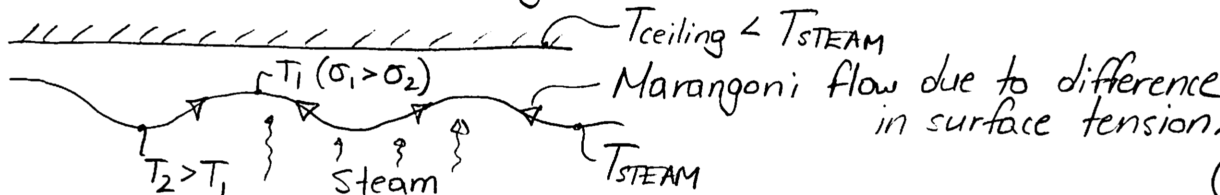
$$F_\sigma \sim F_g \text{ (at departure)} \Rightarrow \sigma \cancel{\pi} \cancel{\lambda_T} \sim (\rho_f-\rho_g) g \cancel{\pi} \tfrac{4}{3} \lambda_T^{\cancel{3}\,2}$$

$$\boxed{\lambda_T \sim \left[\frac{\sigma}{(\rho_f-\rho_g)g}\right]^{1/2}}$$

Aside: $\dfrac{F_g}{F_\sigma} = \dfrac{(\rho_f-\rho_g) g \cancel{\pi} \lambda_T^{\cancel{3}\,2}}{\sigma \cancel{\pi} \cancel{\lambda_T}}$

Bond Number $= \dfrac{\Delta\rho g L^2}{\sigma} = Bo$

Aside: Do you ever notice in the shower, when you look up at the ceiling, that the thin film of condensed water doesn't break up? There are droplets, but the film is intact. This is also due to a Rayleigh-Taylor instability, but Marangoni stresses hold the film together:



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Using Taylor stability analysis, Zuber showed that:

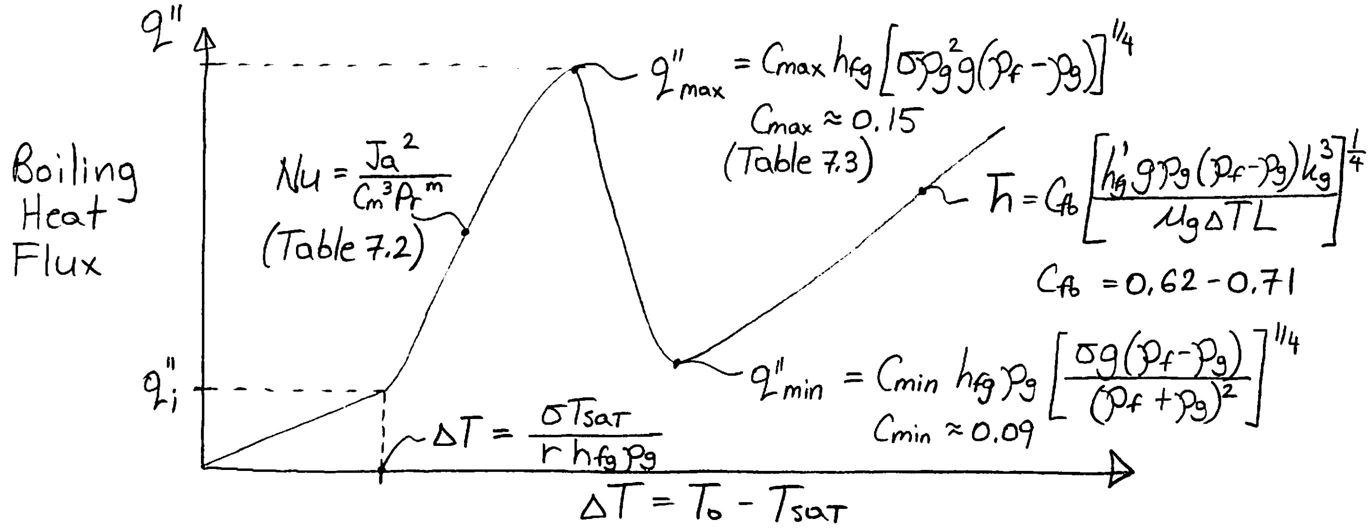
$$\boxed{q''_{min} = C_{min}\,\rho_g h_{fg}\left[\frac{\sigma g(\rho_f-\rho_g)}{(\rho_f+\rho_g)^2}\right]^{1/4}}$$

Where: $C_{min} = 0.09$ for large horizontal surfaces, and

$$C_{min} = 0.0464\left[\frac{18}{L^{*2}(2L^{*2}+1)}\right]^{1/4}$$; for horizontal cylinders $L^* = R/\lambda_T$; R = cylinder radius.

Note, ρ_g and ρ_f are evaluated at T_{sat}.

We now have a complete picture of the boiling phenomena of a stationary fluid. If we re-draw our pool boiling curve for a flat horizontal large surface, we would have:



Some Notes

1) The transition regieme is notoriously difficult to model analytically or numerically. It is characterized by unstable or partial film boiling. At any point the conditions can oscillate between film and nucleate boiling, analogous to the transition Reynolds number in pipe flows ($2300 < Re_d < 10\,000$), where flow oscillates between laminar & turb.

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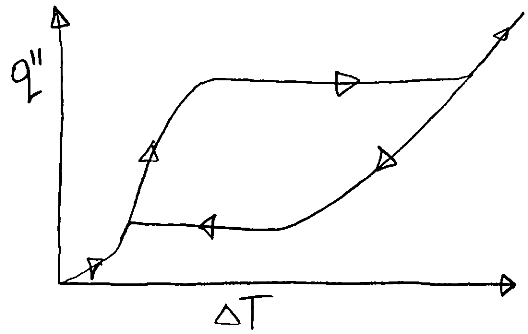
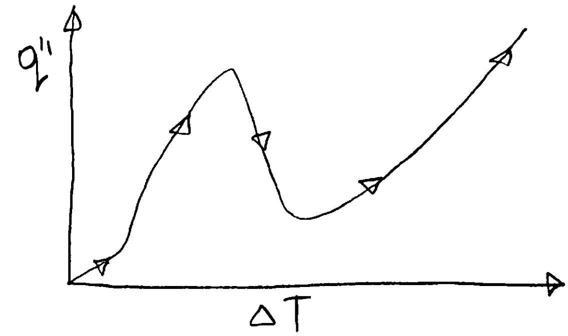
2) The transition region was not observed by Nukiyama since he controled q'' and measured ΔT. If the experiment was set up to control ΔT and measure q'', then the transition region would be observed and no hysteresis would exist.



Control q'', measure ΔT

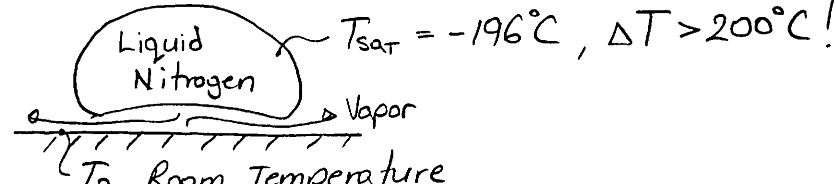


Control ΔT, measure q''

3) Note, in most boiling applications in industry (i.e. industrial power plants, nuclear power, etc...), q'' is the variable input and ΔT cannot be controled. Hence, designers design the system so that q''_{max} or q''_{CHF} is never exceeded. This is why it's termed the critical heat flux, since if $q'' > q''_{CHF}$, the system will melt down. This is what happened at Chernobyl.

$$\boxed{q''_{CHF} = q''_{max} > q''} \quad \text{or} \quad \boxed{q'' \lesssim C_{max} h_{fg} \left[\sigma \rho_g^2 g(\rho_f - \rho_g)\right]^{1/4}}$$

4) q''_{min} is sometimes called the Leidenfrost point or effect, after Johann Leidenfrost. A great demonstration of this is the apparent levitation of liquid nitrogen droplets when spilled on a lab surface. The superheat (ΔT) is so high that the droplet undergoes film boiling and evaporates very slowly:



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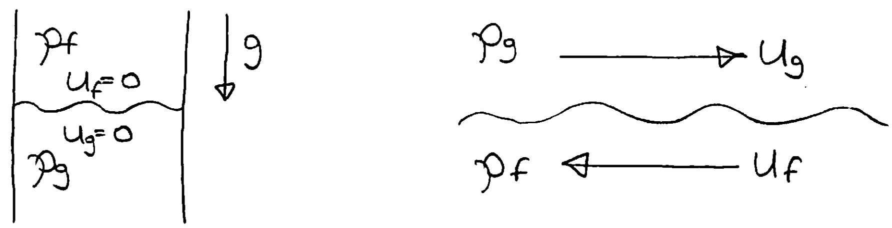
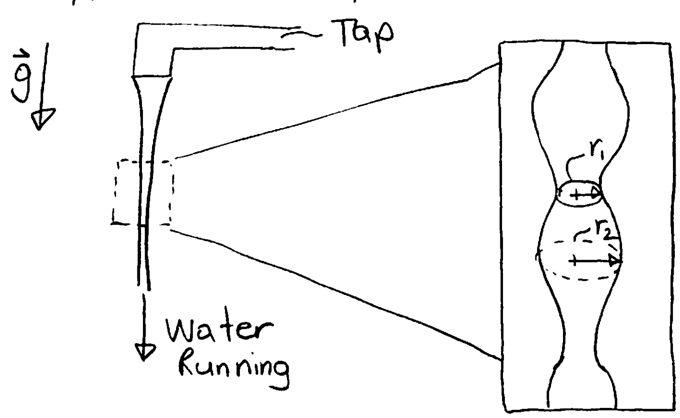
5) The difference in the instability mechanisms for q''_{min} and q''_{max} is that Helmholtz instabilities are shear driven due to a velocity of the gas phase. Taylor instability is surface tension driven and velocity is zero of both phases, with the denser phase on top of the less dense counterpart.



Rayleigh - Taylor   Kelvin - Helmholtz

For more details, consult a graduate level fluid dynamics textbook or class.

6) For your interest only: a common plate to observe the instabilities we've been discussing is your kitchen tap. At low enough flow rates, we observe a Rayleigh-Taylor type instability called the Rayleigh-Plateau instability.

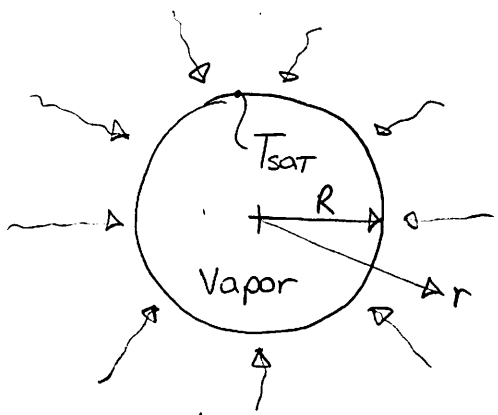


Since $r_1 < r_2$, the surface tension force $(\sim 2\pi r\sigma)$ is larger and will tend to squeeze fluid to the r_2 zones. This causes droplet formation and breakup. Initiates when the fluid stream is very thin, $F_\sigma \uparrow$.

In reality, this is a simplified view since there are other curvature effects that need consideration but nevertheless it is a great instability demonstration in fluid mechanics.

<u>Homogeneous Boiling</u> (Bubble Growth in a Superheated Liquid)

So far, we have only focused on boiling of fluids on solid substrates (heterogeneous). What if no surface exists and boiling occurs inside the superheated fluid?



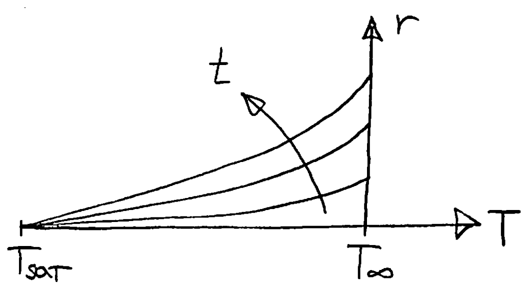
$T_\infty > T_{SAT}$

$P_g \approx P_f$

<u>Assuming:</u> $\frac{\delta_T}{R} \ll 1$ (flat plate model)

$P_g \approx P_f$ (large bubble)

We can treat this as a semi-infinite conduction problem



⇒ The surrounding fluid cools and gives its energy to the bubbe interface (T_{SAT}) for evaporation.

$$q'' = \frac{k_f \Delta T}{\sqrt{\pi \alpha_f t}} = \frac{1}{\sqrt{\pi}} \frac{(k \rho C_p)_f^{1/2} \Delta T}{\sqrt{t}}$$

By an energy balance:

$$\underbrace{q'' 4\pi R^2}_{\text{Energy in}} = \underbrace{\frac{d}{dt}\left(\rho_g \frac{4}{3}\pi R^3\right) h_{fg}}_{\dot{m} h_{fg}} = \rho_g \frac{4}{3}\pi R^2 h_{fg} \frac{dR}{dt}$$

Solving for $\frac{dR}{dt}$ explicity: $\left[\frac{1}{\sqrt{\pi}} \frac{(k\rho C_p)_f^{1/2}}{\rho_g h_{fg}} \Delta T\right] \frac{1}{\sqrt{t}} = \frac{dR}{dt}$

Integrating: $\boxed{R(t) = \frac{2}{\sqrt{\pi}} \frac{(k\rho C_p)_f^{1/2}}{\rho_g h_{fg}} \Delta T \sqrt{t}}$

Note we can make our solution simpler by multiplying by $\frac{C_{p,f}}{C_{p,f}}$

$$R = \frac{2}{\sqrt{\pi}} \frac{(k\rho C_p)_f^{1/2}}{\rho_g C_{p,f}} \left(\frac{C_{p,f}\Delta T}{h_{fg}}\right)\sqrt{t}$$

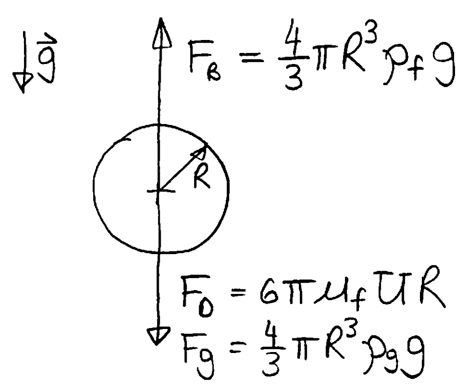
$$= \frac{2}{\sqrt{\pi}} \left(\frac{k_f}{C_{p,f}\rho_f}\right)^{1/2} \left(\frac{Ja\,\rho_f}{\rho_g}\right)\sqrt{t}$$

$$\boxed{R(t) = \frac{2}{\sqrt{\pi}} \cdot Ja^* \sqrt{\alpha_f t}}\ , \text{ where } \boxed{Ja^* = \frac{C_p \Delta T}{h_{fg}} \cdot \frac{\rho_f}{\rho_g}}$$ ⇒ Modified Jacob number.

Note, the above solution is valid for <u>diffusion controled</u> bubble growth, since we assume everything is in quazi-equilibrium. We assume no energy is required to move the liquid as the bubble grows. If the growth is fast, inertia must be accounted for with flow work. This latter case is called <u>inertially controled</u> bubble growth and was solved by Rayleigh in the 1800's.

END OF LECTURE 24

<u>Aside</u>: To give you an example of flow work, lets do a quick calculation of the speed of a bubble rising in water, assuming Stokes' flow ($Re < 0.1$)



Force balance at terminal speed:

$$F_B = F_g + F_D$$

$$\frac{\cancel{4}^2}{3}\cancel{\pi}R^{\cancel{3}2}\rho_f g = \frac{\cancel{4}^2}{3}\cancel{\pi}R^{\cancel{3}2}\rho_g g + \cancel{6}^3\cancel{\pi}\mu_f U \cancel{R}$$

$$\frac{2}{3} R^2(\rho_f - \rho_g) g = 3\mu_f U$$

$$\boxed{U = \frac{2R^2(\rho_f - \rho_g)g}{9\mu_f}}$$

Let's try a $R = 1\text{cm}$ bubbe in water (air is bubble phase, $\rho_g \approx 1.2$)

$$U = \frac{2(0.01)^2(1000 - 1.2)(9.81)}{9(8.94 \times 10^{-4})} \approx 243.8\ \text{m/s}!$$ Very large and not physically accurate.

We haven't included flow work to move water, hence U is very large!