**Thermal Radiation**

So far, we've dealt with the following:

- **Conduction**: Heat transfer by a temperature gradient in a medium.
- **Convection**: - no medium required (can propagate in vacuum)
- **Radiation**: - is a form of energy emitted by all matter at a finite temperature.

**Applications:**
1) Global energy balance → earth receives all of its energy from the sun via radiation.
2) HVAC → Radiative heaters. Work well in areas where lots of "air changes" or opening of doors to the outside occurs, i.e. entrance ways to a theater, outside, warehouses.

**Classic HVAC ⇒ Convection**

- Cold air currents remove heat quickly & you feel cold.

**Radiative Heater**

- Radiative heater keeps heating you irrespective of air currents

**Theory:**

1. Electromagnetic Theory (Maxwell) ⇒ waves

\[
C = \lambda U = \frac{3.0 \times 10^8 \text{ m/s}}{n}, \quad \text{where} \quad n = \text{index of refraction} \\
U = \frac{C}{\lambda} \quad \text{C = speed of light}
\]
2. Quantum Theory (Planck) => photons, packets of energy

\[ e = \frac{hc}{\lambda} \]

where:
- \( e \) = energy of a photon [J]
- \( c \) = speed of light [m/s]
- \( \lambda \) = wavelength of light [m]
- \( h = 6.6256 \times 10^{-34} \text{ J s} \)

These two theories came to a conflict in early 1900's. The classical theory predicted an infinite "radiative energy" of a body at a finite temperature.

For higher frequency modes, more can fit in our finite box than lower frequency modes. Using analysis of standing waves, we can show that:

\[ \frac{\text{# modes per unit wavelength}}{\text{Cavity volume}} = \frac{8\pi}{\lambda^4} \]

Using equipartition of energy, the energy per mode is \( k_b T \).

\[ E_{\text{mode}} = \frac{1}{2} k_b T \times 2 \text{ polarizations/mode} \]

where: \( k_b = 1.38 \times 10^{-23} \)
So: \( U(T) = -\int_{0}^{u_{1}} du = -\int_{0}^{\lambda} \frac{8\pi k_{B} T}{\lambda^4} d\lambda \rightarrow \infty \) as \( \lambda \rightarrow 0 \)

This was called the ultraviolet catastrophe. As the wavelength reached the UV levels, energy shoots to infinity.

This was resolved by Planck. The main difference is:

1) Classical view assumed all modes were equally probable to be occupied. Adding a small amount of energy to any mode is possible.

2) Planck's assumption that energy is quantized (add the energy of a whole photon, or don't add it at all) treats the photons like particles. As we learned in kinetic theory, the probability of finding a particle at the average kinetic energy is higher than finding it at a much higher energy state. Hence, it is harder to find a photon far away from the average thermal energy \( k \) hence the probability of UV photons is low.

\[ T_1 > T_2 > T_3 > T_4 \]

The UV Catastrophe
Rayleigh-Jeans Law

\[ E_{b, \lambda} = \text{monochromatic emissive power} \]
Planck derived his formula to be:

\[ e_b,\lambda = \frac{2\pi \hbar c^2 \lambda^{-5}}{e^{\frac{\hbar}{kT \lambda}} - 1} = \frac{C_1 \lambda^{-5}}{e^{\frac{C_2}{\lambda T}} - 1} \]

Dividing through by \( T^5 \)

\[ \frac{e_b,\lambda}{T^5} = \frac{C_1 (\lambda T)^{-5}}{e^{\frac{C_2}{\lambda T}} - 1} \]

\( C_1 = 2\pi \hbar c^2 = 3.742 \times 10^8 \text{ W m}^4/\text{m}^2 \)
\( C_2 = \hbar c/k_B = 1.4389 \times 10^4 \text{ km K} \)

\[ (\lambda T)_{\text{max}} = 2898 \text{ km K} \]

\( \Rightarrow \) Wien's displacement law.

Visible range: 0.4 - 0.7 \( \mu \text{m} \) (blue to red)

Thermal radiation: 0.1 - 100 \( \mu \text{m} \)

So as energy increases (\( T^4 \)), wavelength is reduced and more of the emissive spectrum shifts to the visible range.

Example: hot iron bar \( \Rightarrow \) goes from no color \( \Rightarrow \) red \( \Rightarrow \) orange \( \Rightarrow \) yellow \( \Rightarrow \) white.

Note, if we want total emissive power:

\[ e_b = \int_0^\infty e_b,\lambda \, d\lambda = T^5 C_1 \int_0^\infty \frac{(\lambda T)^{-5}}{e^{\frac{C_2}{\lambda T}} - 1} \, d\lambda \]

\[ = T^4 \left[ C_1 \int_0^\infty \frac{n^{-5}}{e^{\frac{C_2}{n}} - 1} \, dn \right], \quad n = \lambda T \]

\( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \)
So our total emissive power is:

\[ e_b = \sigma T^4 \]

It is important to note, that the previous analysis is valid for "blackbodies." These are ideal radiators:

1) They absorb all incident (incoming) radiation
2) Emission of radiation is independent of direction (diffuse)

\[ \sim \text{emission} \]

8) At a given temp. (T) and wavelength (\( \lambda \)), no surface can emit more energy.

Experimentally, a blackbody can be simulated by:

incident radiation

- all incident radiation is absorbed (multiple bounces)
- emitted radiation independent of direction
- the small hole is the blackbody.

For non-blackbody surfaces, we can define a monochromatic emissivity \( E_\lambda \):

\[ e_\lambda = E_\lambda e_b,\lambda \]

\[ e = \int_0^\infty E_\lambda e_b,\lambda d\lambda = E e_b \]
\[ E = \frac{\int_{\lambda_0}^{\infty} e_\lambda e_{\lambda,\nu} d\lambda}{\int_{\lambda_0}^{\infty} e_{\lambda,\nu} d\lambda} \]

\[ \Rightarrow \text{Spectrally averaged emissivity.} \]

For a real surface, we also have reflection (\( \rho \)) and absorption (\( \alpha \))

\[ \text{Incoming Radiation} \quad H_\lambda \quad \text{Outgoing Radiation} \quad \rho_\lambda H_\lambda \]

\[ \alpha_\lambda H_\lambda \]

\[ \text{From an energy balance:} \quad \rho_\lambda + \alpha_\lambda = 1 \]

\[ \text{In thermal equilibrium:} \quad E_\lambda = \alpha_\lambda \quad \Rightarrow \text{Kirchhoff's Law} \]

Note, this is valid for diffuse surfaces, otherwise \[ E_{\lambda,\theta} = \alpha_{\lambda,\theta} \]

Similarly for absorption:

\[ \alpha = \frac{\int_{\lambda_0}^{\infty} \alpha_\lambda H_\lambda d\lambda}{\int_{\lambda_0}^{\infty} H_\lambda d\lambda} \]

\[ \Rightarrow \text{Spectrally averaged absorption} \]

Naturaly from Kirchhoff's law, we can say:

1) Good absorbers are good emitters \( \Rightarrow \text{experimentally established.} \)
2) Poor absorbers are poor emitters

This is why high tech thermal blankets are made of reflective metallic coatings, since poor absorbers are poor emitters, and will keep you warm.

Also note, the above assumes transmission is zero. If not:

\[ \alpha_\lambda + \rho_\lambda + T_\lambda = 1 \]