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Thermal Radiation

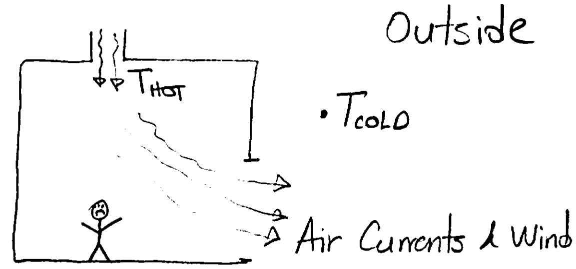
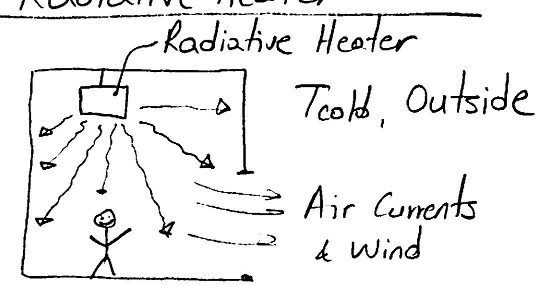
So far, we've dealt with the following:

Conduction, Convection } Heat transfer by a temperature gradient in a medium.

Radiation }
- no medium required (can propagate in vacuum)
- is a form of energy emitted by all matter at a finite temperature.

Applications:

1) Global energy balance → earth receives all of it's energy from the sun via radiation.
2) HVAC → Radiative heaters. Work well in areas where lots of "air changes" or opening of doors to the outside occurs, i.e. entrance ways to a theater, outside, wearhouses.

Classic HVAC ⇒ Convection	Radiative Heater
	
- Cold air currents remove heat quickly & you feel cold.	- Radiative heater keeps heating you irrespective of air currents

Theory:

1. Electromagnetic Theory (Maxwell) ⇒ waves

$$\boxed{c = \lambda \nu = \frac{3.0 \times 10^{8}\ m/s}{n}}$$

, where n = index of refraction

$\nu = \frac{c}{\lambda}$

c = speed of light

↳ Used for Rayleigh-Jeans Law.

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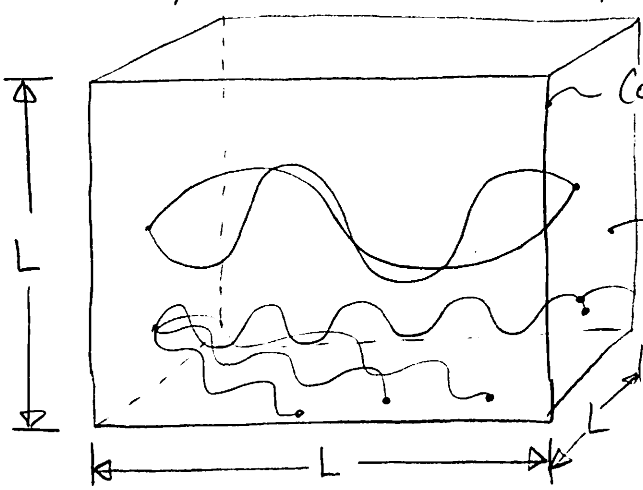
(2) Quantum Theory (Planck) $\Rightarrow$ photons, packets of energy

$$\boxed{e = \frac{hc}{\lambda} = h\nu}$$, where e = energy of a photon [J]
c = speed of light [m/s]
λ = wave length of light [m]
$h = 6.6256 \times 10^{-34}$ J·s

These two theories came to a conflict in early 1900's. The classical theory predicted an infinite "radiative energy" of a body at a finite temperature.



For higher frequency modes, more can fit in our finite box than lower frequency modes. Using analysis of standing waves, we can show that:

$$\boxed{\frac{\#\text{modes per unit wavelength}}{\text{Cavity volume}} = \frac{8\pi}{\lambda^4}}$$

Using equipartition of energy, the energy per mode is $k_B T$.

$$E_{mode} = \frac{\frac{1}{2} k_B T \cdot (2)}{\text{degree of freedom}}$$ ← 2 polarizations/mode

, where: $k_B = 1.38 \times 10^{-23}$

$$\boxed{\frac{du}{d\lambda} = \frac{8\pi k_B T}{\lambda^4}}$$ $\Rightarrow$ Energy per unit volume per unit wavelength

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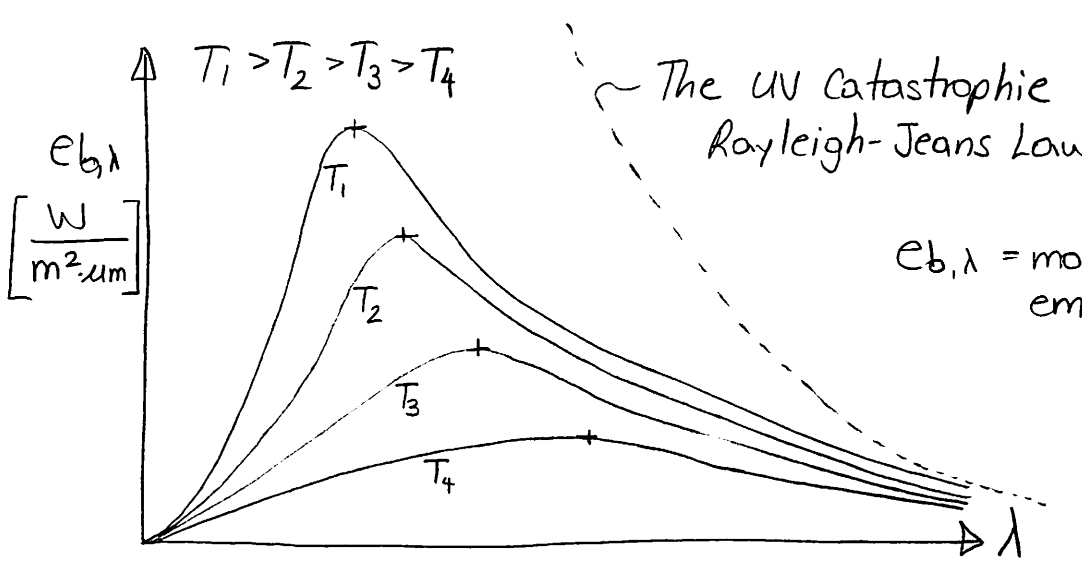
So: $U(T) = \int_0^{u_\lambda} du = \int_0^{\lambda} \frac{8\pi k_B T}{\lambda^4} d\lambda \longrightarrow \infty$ as $\lambda \to 0$

This was called the ultraviolet catastrophie. As the wavelengths reached the UV levels, energy shoots to infinity.

This was resolved by Planck. The main difference is:

1) Classical view assumed all modes were equally probable to be occupied. Adding a small amount of energy to any mode is possible.

2) Planck's assumption that energy is quantized (add the energy of a whole photon, or don't add it at all) treats the photons like particles. As we learned in kinetic theory, the probability of finding a particle at the average kinetic energy is higher than finding it at a much higher energy state. Hence, it is harder to find a photon far away from the average thermal energy & hence the probability of UV photons is low.



$e_{b,\lambda}$ = monochromatic emissive power

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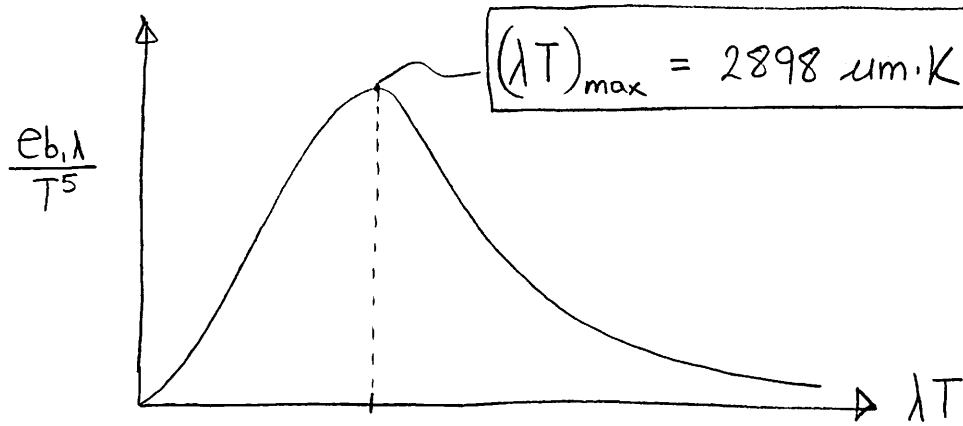
Planck derrived his formula to be:

$$e_{b,\lambda} = \frac{2\pi h c^2 \lambda^{-5}}{e^{\frac{ch}{k_B \lambda T}} - 1} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

Dividing through by T^5

$$\boxed{\frac{e_{b,\lambda}}{T^5} = \frac{C_1 (\lambda T)^{-5}}{e^{C_2/\lambda T} - 1}} \quad , \quad \begin{aligned} C_1 &= 2\pi h c^2 = 3.742 \times 10^8 \ W um^4/m^2 \\ C_2 &= hc/k_B = 1.4389 \times 10^4 \ um \cdot K. \end{aligned}$$



$$\boxed{(\lambda T)_{max} = 2898 \ um \cdot K} \Rightarrow \text{Wien's displacement law.}$$

Visible range: 0.4 - 0.7 um (blue to red)
Thermal radiation: 0.1 - 100 um

So as energy increases ($T \uparrow$), wavelength is reduced and more of the emissive spectrum shifts to the visible range.

Example: hot iron bar $\Rightarrow$ goes from no color $\to$ red $\to$ orange $\to$ yellow $\to$ white.

Note, if we want total emissive power:

$$e_b = \int_0^\infty e_{b,\lambda} \, d\lambda = T^5 C_1 \int_0^\infty \frac{(\lambda T)^{-5}}{e^{C_2/\lambda T} - 1} \, d\lambda$$

$$= T^4 \underbrace{\left[C_1 \int_0^\infty \frac{z^{-5}}{e^{C_2/z} - 1} \, dz \right]}_{\sigma = 5.67 \times 10^{-8} \ W/m^2 K^4} , \quad z = \lambda T$$

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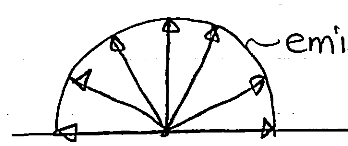
So our total emissive power is:

$$\boxed{e_b = \sigma T^4}$$

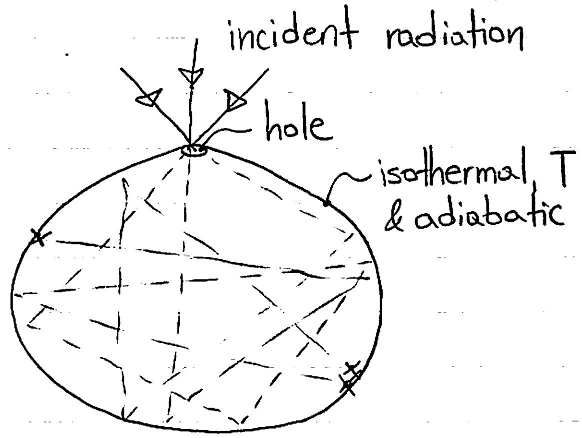
It is important to note, that the previous analysis is valid for "blackbodies." These are ideal radiators:

1) They absorb all incedent (incoming) radiation
2) Emission of radiation is independent of direction (diffuse)



3) At a given temp. (T) and wavelength (λ), no surface can emit more energy.

Experimentally, a blackbody can be simulated by:



- all incident radiation is absorbed (multiple bounces)
- emitted radiation independent of direction
- the small hole is the blackbody.

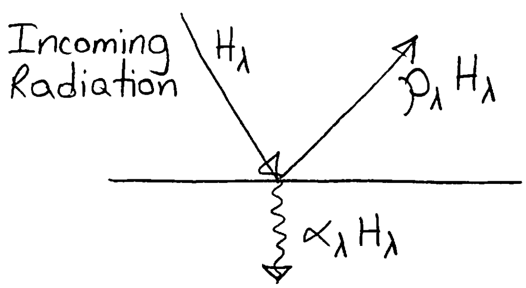
For non-blackbody surfaces, we can define a monochromatic emissivity ε_λ:

$$e_\lambda = \varepsilon_\lambda e_{b,\lambda}$$

$$e = \int_0^\infty \varepsilon_\lambda e_{b,\lambda} d\lambda = \varepsilon e_b$$

$$\boxed{\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda e_{b,\lambda}\, d\lambda}{\int_0^\infty e_{b,\lambda}\, d\lambda}} \Rightarrow \text{Spectrally averaged emissivity.}$$

For a real surface, we also have reflection (ρ) and absorption (α)



From an energy balance:

$$\boxed{\rho_\lambda + \alpha_\lambda = 1}$$

In thermal equilibrium:

$$\boxed{\varepsilon_\lambda = \alpha_\lambda} \Rightarrow \text{Kirchhoff's Law}$$

Note, this is valid for diffuse surfaces, otherwise $\boxed{\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}}$

Similarly for absorption:

$$\boxed{\alpha = \frac{\int_0^\infty \alpha_\lambda H_\lambda\, d\lambda}{\int_0^\infty H_\lambda\, d\lambda}} \Rightarrow \text{Spectrally averaged absorption}$$

Naturally from Kirchhoff's law we can say:

1) Good absorbers are good emitters
2) Poor absorbers are poor emitters

} experimentally established.

This is why high tech thermal blankets are made of reflective metallic coatings, since poor absorbers are poor emitters, and will keep you warm.

Also note, the above assumes transmission is zero. If not:

$$\boxed{\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1}$$

END OF LECTURE 25