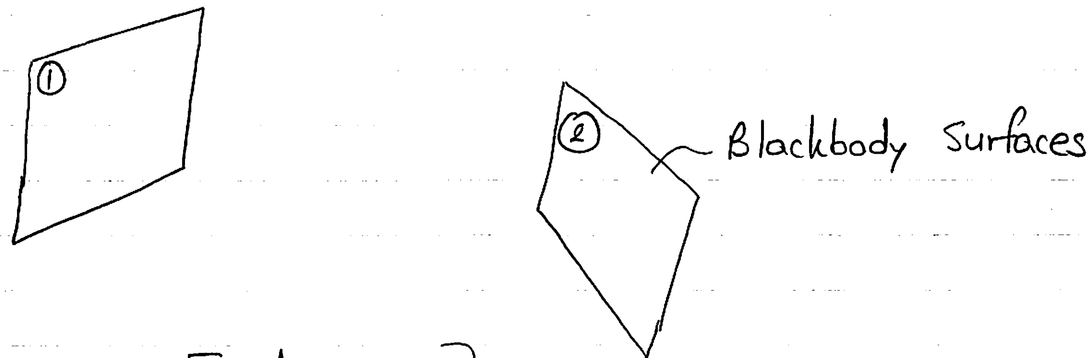
Nenad Miljkovic

View Factors and Blackbody Exchange



$$q_{1\to 2} = F_{12} A_1 e_{b1}$$
$$q_{2\to 1} = F_{21} A_2 e_{b2}$$

F_{ab} = view factor = fraction of radiation leaving surface "a" and reaching surface "b".

So our net exchange is:

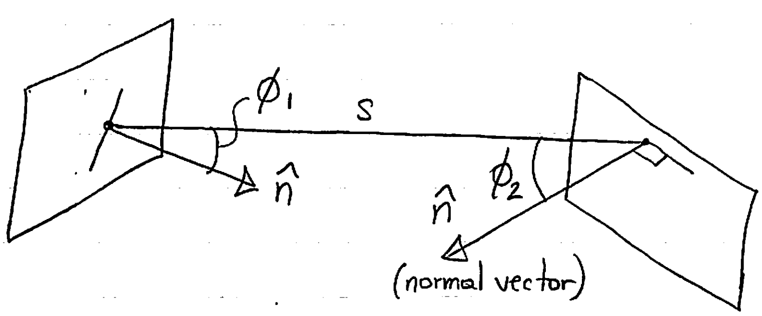
$$q_{12} = q_{1\to 2} - q_{2\to 1} = F_{12} A_1 e_{b1} - F_{21} A_2 e_{b2}$$

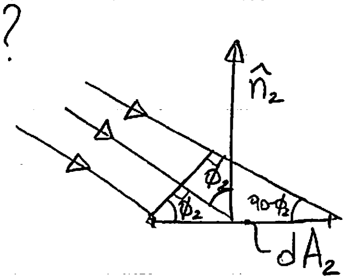
If $T_1 = T_2$, $e_{b1} = e_{b2}$

$$q_{12} = e_b (A_1 F_{12} - A_2 F_{21}) = 0$$ (no net energy transfer between bodies at same T)

$$\therefore \boxed{A_1 F_{12} = A_2 F_{21}}$$

But what is our fraction F_{12} or F_{21}?





$$A_1 F_{12} = B \int_{A_1}\int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{s^2}\, dA_1 dA_2$$; B = constant

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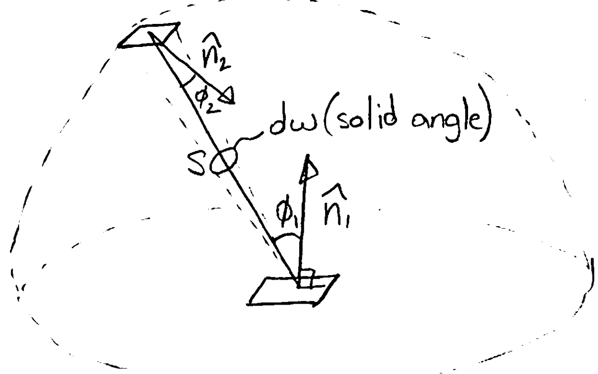
Aside:

Think of the previous expression in terms of small differential areas:



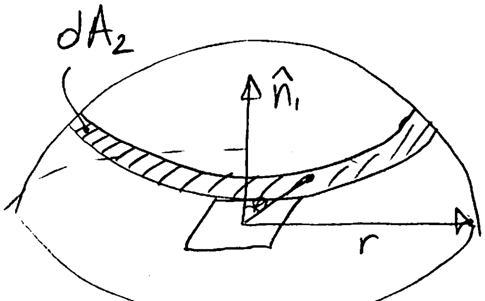
$$d\omega = \frac{\cos\phi_2\, dA_2}{S^2} \quad \text{(Solid Angle)}$$

$$e_{OUT} = I_{OUT}\cos\phi_1\, dA_1\, d\omega$$

$$I_{OUT} = \frac{\sigma T^4}{\pi} \quad \text{(Intensity)}$$

You can solve the rest. Try it!

Going a step further



$$A_1 = dA_1$$
$$S = r$$
$$\phi_2 = 0$$
$$dA_2 = 2\pi r^2 \sin\phi_1\, d\phi_1$$

Substituting back into our relation : $\underbrace{\cancel{A_1}}_{dA_1} F_{12} = B \int_{A_1}\int_{A_2} \frac{\cos\phi_1\cos\phi_2}{S^2}\, \cancel{dA_1}\, dA_2$

$$F_{12} = B \cdot 2\pi \int_0^{\pi/2} \cos\phi_1 \sin\phi_1\, d\phi_1$$

$$F_{12} = 1$$

$$1 = B \cdot 2\pi \int_0^{\pi/2} \sin\phi_1\, d(\sin\phi_1) = B\, 2\pi \left.\frac{(\sin\phi_1)^2}{2}\right|_0^{\pi/2} = \pi B = 1$$

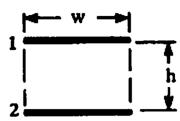
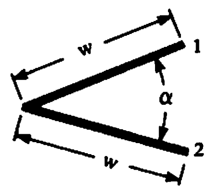
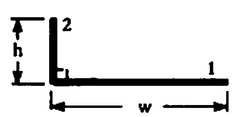
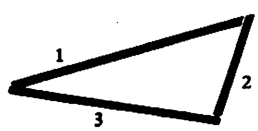
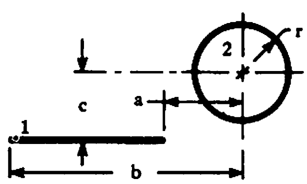
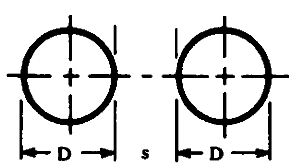
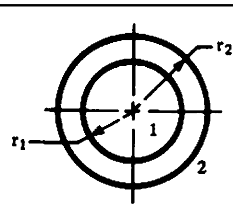
$$\boxed{B = \frac{1}{\pi}}$$

So our view factor becomes:

$$\boxed{A_1 F_{12} = A_2 F_{21} = \frac{1}{\pi}\int_{A_1}\int_{A_2} \frac{\cos\phi_1\cos\phi_2}{S^2}\, dA_1\, dA_2}$$

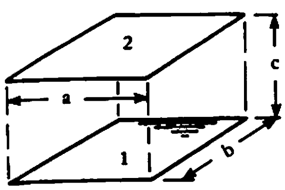
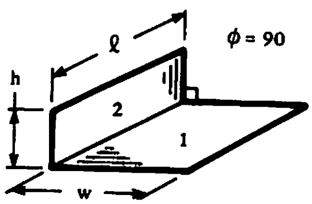
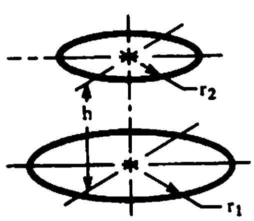
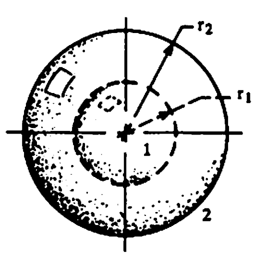
$\Rightarrow$ Look up in Mills Table 6.1, pg. 541-543.

Table 10.2 View factors for a variety of two-dimensional configurations (infinite in extent normal to the paper)

Configuration		*Equation*
1.		$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$
2.		$F_{1\text{-}2} = F_{2\text{-}1} = 1 - \sin(\alpha/2)$
3.		$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$
4.		$F_{1\text{-}2} = (A_1 + A_2 - A_3)/2A_1$
5.		$F_{1\text{-}2} = \frac{r}{b-a}\left[\tan^{-1}\frac{b}{c} - \tan^{-1}\frac{a}{c}\right]$
6.		Let $X = 1 + s/D$. Then: $F_{1\text{-}2} = F_{2\text{-}1} = \frac{1}{\pi}\left[\sqrt{X^2 - 1} + \sin^{-1}\frac{1}{X} - X\right]$
7.		$F_{1\text{-}2} = 1$, $F_{2\text{-}1} = \frac{r_1}{r_2}$, and $F_{2\text{-}2} = 1 - F_{2\text{-}1} = 1 - \frac{r_1}{r_2}$

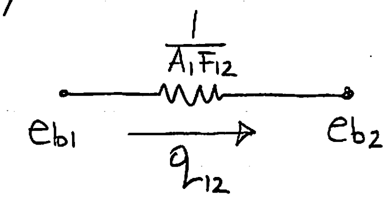
Adapted From Lienhard & Lienhard "A Heat Transfer Textbook" (2012)

Table 10.3 View factors for some three-dimensional configurations

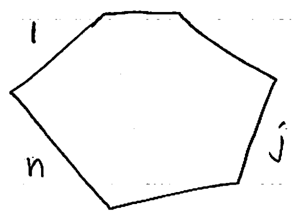
Configuration	*Equation*
1. 	Let $X = a/c$ and $Y = b/c$. Then: $$F_{1\text{-}2} = \frac{2}{\pi XY}\left\{ \ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} - X\tan^{-1}X - Y\tan^{-1}Y + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}}\right\}$$
2. 	Let $H = h/\ell$ and $W = w/\ell$. Then: $$F_{1\text{-}2} = \frac{1}{\pi W}\left\{ W\tan^{-1}\frac{1}{W} - \sqrt{H^2+W^2}\tan^{-1}\left(H^2+W^2\right)^{-1/2} + H\tan^{-1}\frac{1}{H} + \frac{1}{4}\ln\left\{\left[\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\right] \times \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2}\left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right\}$$
3. 	Let $R_1 = r_1/h$, $R_2 = r_2/h$, and $X = 1 + \left(1 + R_2^2\right)/R_1^2$. Then: $$F_{1\text{-}2} = \frac{1}{2}\left[X - \sqrt{X^2 - 4(R_2/R_1)^2}\right]$$
4. 	Concentric spheres: $$F_{1\text{-}2} = 1, \quad F_{2\text{-}1} = (r_1/r_2)^2, \quad F_{2\text{-}2} = 1 - (r_1/r_2)^2$$

Now we can formulate our analysis as:

$$q_{12} = \underbrace{A_1 F_{12} (e_{b1} - e_{b2})}_{A_1 F_{12} e_{b1} - A_2 F_{21} e_{b2}} \iff$$



If we have an enclosure: (Radiation exchange inside a body)

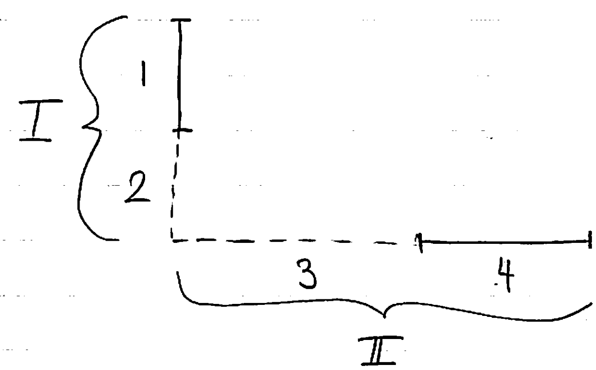


$$\boxed{\sum_{j=2}^{n} F_{ij} = 1}$$

Note, only true for flat or convex surfaces. If concave

$\sum_{j=2}^{n} F_{ij} \neq 1$ since $F_{11} \neq 0$ (Surface sees itself).

<u>Example</u> Consider the following: (Solve for F_{14})



$$A_1 F_{14} = A_I F_{I4} - A_2 F_{24}$$

$$\cancel{A_I} F_{I4} = \cancel{A_I} F_{I\,II} - \cancel{A_I} F_{I3} \Rightarrow F_{I4} = \underbrace{F_{I\,II} - F_{I3}}$$

$$\cancel{A_2} F_{24} = \cancel{A_2} F_{2II} - \cancel{A_2} F_{23} \Rightarrow F_{24} = \underbrace{F_{2II} - F_{23}}$$

Our charts have these factors: $F_{I\,II}, F_{I3}, F_{2II}, F_{23}$ ← Can easily look these up.

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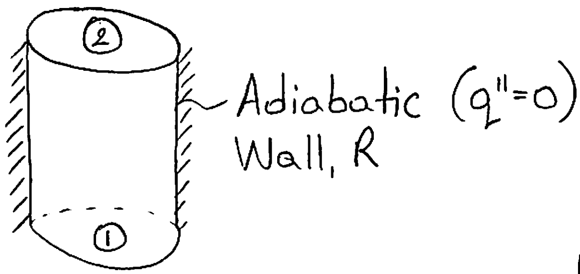
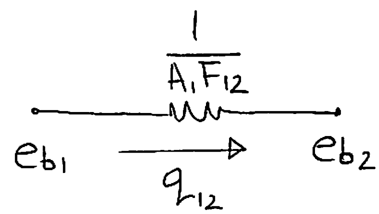
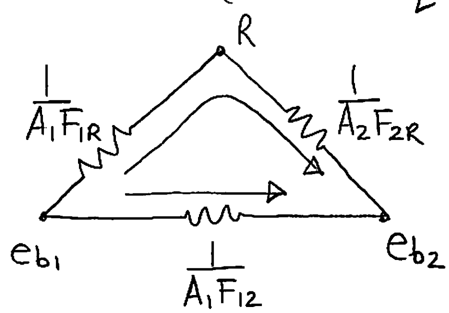
Adiabatic Surfaces (no heat transfer through them)



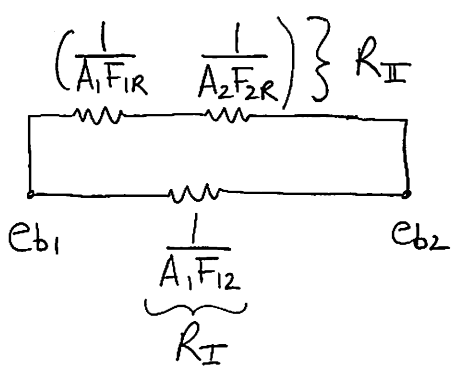
Previously we had:



Now we have (due to $q''=0$)



To solve, we now have:



$$\Rightarrow \quad \frac{1}{R_{TOT}} = \frac{1}{R_I} + \frac{1}{R_{II}}$$

$$= A_1 F_{12} + \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}}}$$

$$\boxed{q_{12} = \frac{e_{b1} - e_{b2}}{R_{TOT}} = (e_{b1} - e_{b2}) \left[A_1 F_{12} + \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}}} \right]}$$

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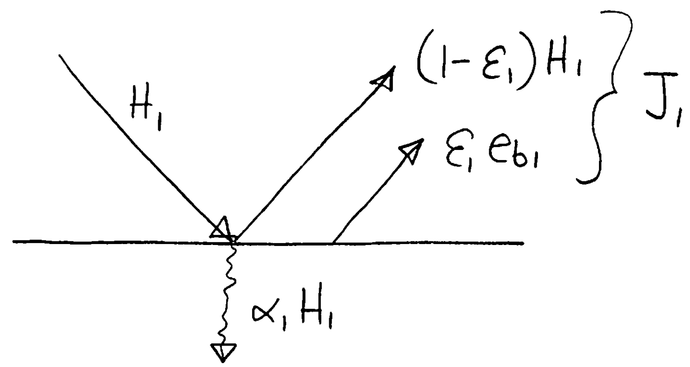
Gray Body Radiation

diffuse & idependent of wavelength

If $\varepsilon \neq 1$, and $\varepsilon \neq f(\lambda, \theta)$, we call this a gray body.
Here, we can assume that $\varepsilon = \alpha$.



Flux of energy that irradiates the surface.

H_1 = irradiance $\left[\frac{W}{m^2}\right]$

J_1 = radiosity $\left[\frac{W}{m^2}\right]$

Total flux of radiat energy away from the surface.

$$J_1 = \underbrace{\underbrace{(1-\varepsilon_1)H_1}_{(1-\alpha_1)H_1}}_{\text{Reflected energy}} + \underbrace{\varepsilon_1 e_{b1}}_{\text{Emitted energy}}$$

Aside: Note, gray body analysis is very usefull in real life since most surfaces don't behave as blackbodies and have some finite emissivity $\varepsilon < 1$ & $\alpha < 1$

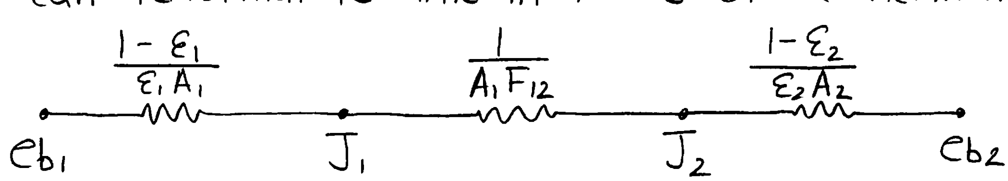
$$H_1 = \frac{J_1 - \varepsilon_1 e_{b1}}{1-\varepsilon_1}$$

$$\frac{q_{1,NET}}{A_1} = (J_1 - H_1) = J_1 - \frac{J_1 - \varepsilon_1 e_{b1}}{1-\varepsilon_1}$$

$$= \frac{J_1(1-\varepsilon_1) - J_1 + \varepsilon_1 e_{b1}}{1-\varepsilon_1} = \frac{\varepsilon_1}{1-\varepsilon_1}(e_{b1} - J_1)$$

$$\boxed{\frac{q_{1,NET}}{A_1} = \frac{\varepsilon_1}{1-\varepsilon_1}(e_{b1} - J_1)}$$ $\Rightarrow$ Net heat flux into surface 1.

We can reformulate this in terms of a thermal resistance



Note, if you set $\varepsilon_1 = \varepsilon_2 = 1$, you get the blackbody solution.

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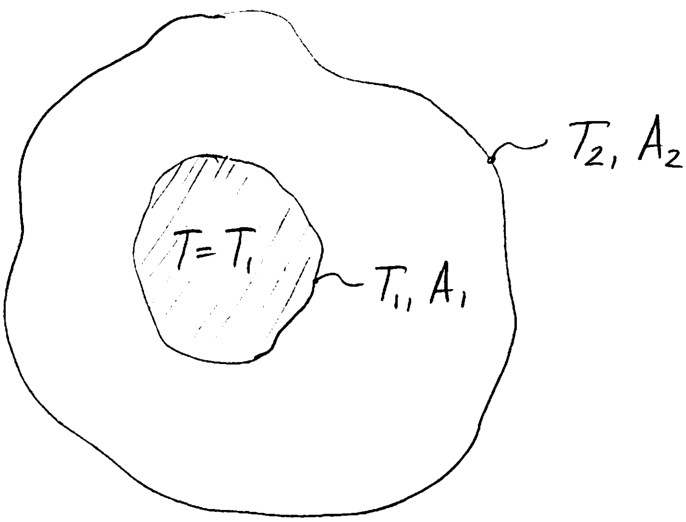
Gray Body Enclosures



By an energy ballance:

$$\cancel{A_1}H_1 = \underbrace{A_2 J_2 F_{21}}_{\cancel{A_1} J_2 F_{12}} + \cancel{A_1} J_1 F_{11}$$

$$H_1 = J_2 F_{12} + J_1 F_{11}$$

↳ Net energy coming to ①

$$\frac{q_{1\to 2,NET}}{A_1} = J_1 - J_2F_{12} - J_1F_{11}$$

$$= \underbrace{(J_1 - J_2)F_{12}}_{\text{Since } (1-F_{11}) = F_{12}}$$

So we have the same solution as before, but assuming that body 2 is much larger than body 1, we get $F_{12} = 1$.

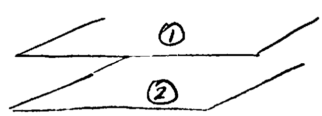
$$q_{1\to 2} = \frac{A_1(e_{b1} - e_{b2})}{\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

① $A_2 >> A_1$; $F_{12} = 1$

$$\boxed{q_{1-2} = A_1 \varepsilon_1 (e_{b1} - e_{b2})}$$ ⇒ Smaller body behaves like a black body.

Physically, this result comes from the fact that the emitted and reflected radiation from ① bounces around multiple times and is absorbed by ② before it can hit ① again. Hence $\varepsilon_2 = 1$. $\alpha_2 = 1$.

② Parallel Plates:



$F_{12} = 1 = F_{21}$
$A_1 = A_2$

$$\boxed{q_{1-2} = \frac{A(e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

END OF LECTURE 26