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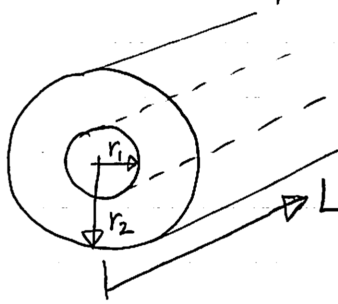
③ Concentric Cylinders

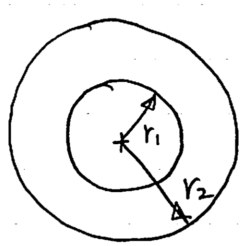


$$F_{12} = 1$$

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$\boxed{q_{1-2} = \frac{2\pi r_1 L\,(e_{b_1} - e_{b_2})}{\frac{1}{\varepsilon_1} + \frac{r_1}{r_2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

④ Concentric Spheres

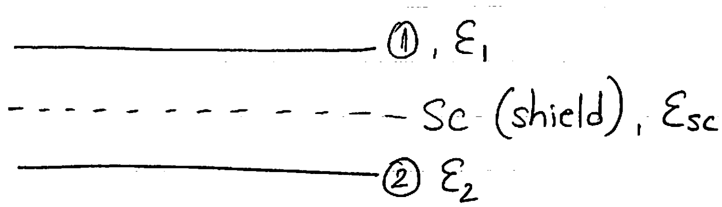


$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad ; \quad F_{12} = 1$$

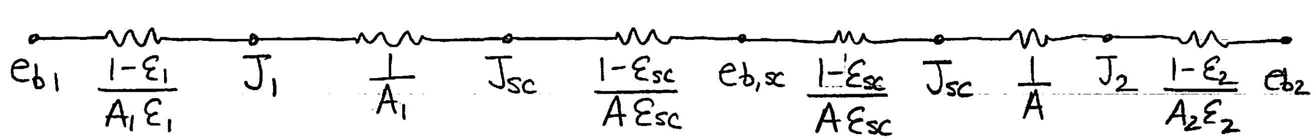
$$\boxed{q_{1-2} = \frac{4\pi r_1^2\,(e_{b_1} - e_{b_2})}{\frac{1}{\varepsilon_1} + \frac{r_1^2}{r_2^2}\left(\frac{1}{\varepsilon_2} - 1\right)}}$$

Radiation Shields

They are often used to reduce heat losses



Drawing out our resistance diagram



$$q_{12} = \frac{A\,(e_{b1} - e_{b2})}{\left(\frac{1}{\varepsilon_1} - 1\right) + 1 + \left(\frac{1}{\varepsilon_2} - 1\right) + 2\left(\frac{1}{\varepsilon_{sc}} - 1\right) + 1}$$

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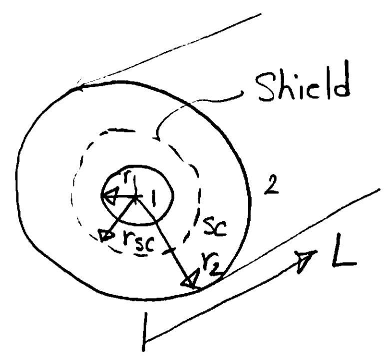
$$\boxed{q_{12} = \frac{A\left(e_{b1}-e_{b2}\right)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{2}{\varepsilon_{sc}}-1\right)}} \Rightarrow \text{For 1 shield}$$

For n shields in series:

$$\boxed{q_{12} = \frac{A\left(e_{b1}-e_{b2}\right)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}+\frac{2n}{\varepsilon_{sc}}-(n+1)}}$$

For <u>cylindrical shields</u> (n of them)

$$\boxed{q_{12} = \frac{2\pi r_1 L\left(e_{b1}-e_{b2}\right)}{\frac{1}{\varepsilon_1}+\frac{r_1}{r_2}\left(\frac{1}{\varepsilon_2}-1\right)+2\left(\frac{1}{\varepsilon_{sc}}-1\right)\sum_{j=1}^{n}\frac{r_1}{r_{sc,j}}}}$$

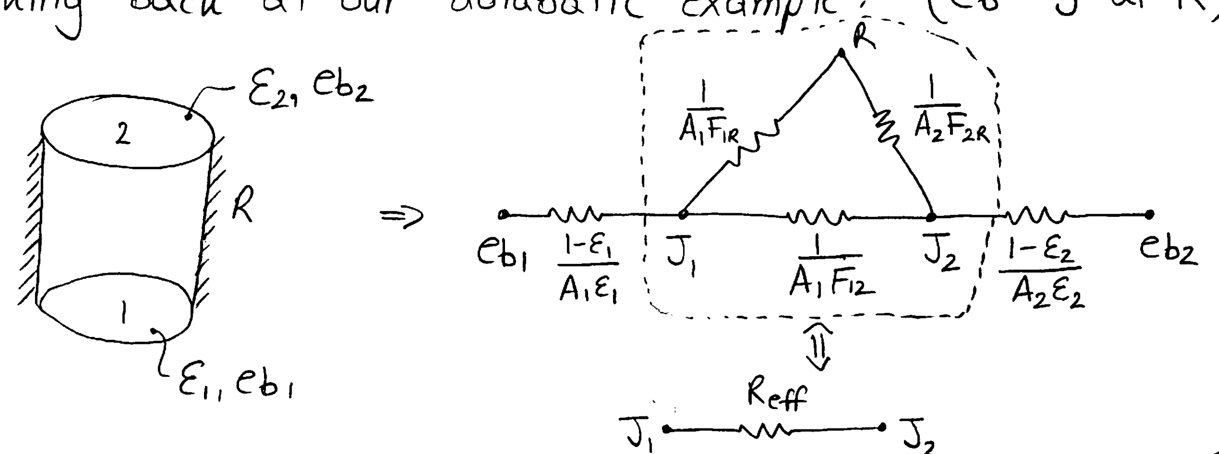


For <u>spherical shields</u> (n of them)

$$\boxed{q_{12} = \frac{4\pi r_1^2\left(e_{b1}-e_{b2}\right)}{\frac{1}{\varepsilon_1}+\frac{r_1^2}{r_2^2}\left(\frac{1}{\varepsilon_2}-1\right)+2\left(\frac{1}{\varepsilon_{sc}}-1\right)\sum_{j=1}^{n}\frac{r_1^2}{r_{sc,j}^2}}}$$

Note, for sherical & cylindrical shields, you want r_{sc} to be close to r_1 since you want the shield to see as little of itself as possible.

Looking back at our adiabatic example: ($e_b = J$ at R)



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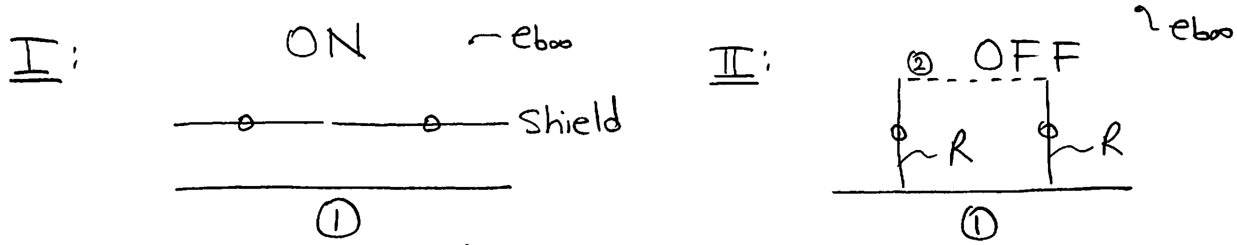
$$\frac{1}{R_{eff}} = \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}}} + A_1 F_{12}$$

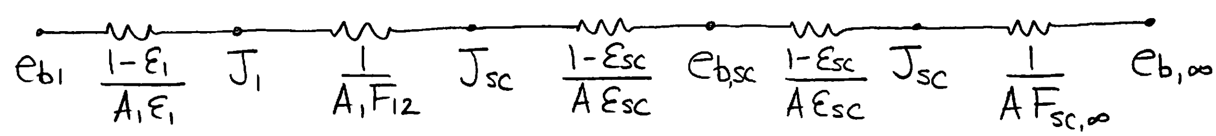
$$\boxed{q_{12} = \frac{e_{b1} - e_{b2}}{\frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + R_{eff}}}$$

⇒ Note, our solution does not depend on ε_R since $e_{bR} = J_R$ or $q'' = 0$ at R.

Example Consider a rotatable radiation shield shown below. Calculate its effectiveness in the on & off positions:

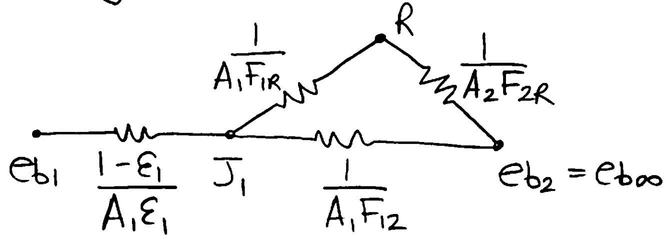


Solving the on case first:



$$q_{1\infty} = \frac{A_1(e_{b1} - e_{b\infty})}{\frac{1}{\varepsilon_1} + \frac{2}{\varepsilon_{sc}} - 1}, \quad F_{sc,\infty} = 1$$

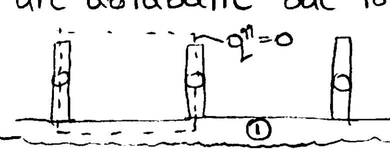
Looking at the OFF case (surfaces are reflective since $q'' = 0$ at the shields)



$$\left.\begin{aligned} F_{12} &= \sqrt{2} - 1 \\ F_{1R} &= 2 - \sqrt{2} \end{aligned}\right\} \text{From tables}$$

$$q_{12} = \frac{A_1(e_{b1} - e_{b\infty})}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}}$$

Aside: Note, the shields in II are adiabatic due to symmetry



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$R_{I} \gg R_{II}$, Assuming $\varepsilon_1 = 0.5$, $\varepsilon_{sc} = 0.1$; R = thermal radiative resistance.

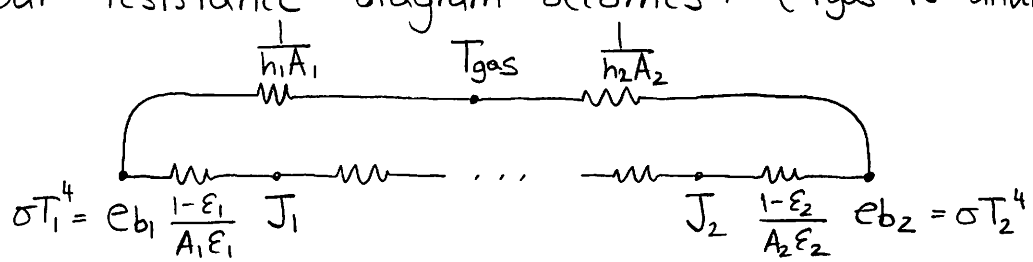
$$\frac{R_I}{R_{II}} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{sc}} - 1}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}} = 13.2$$

So when the shield is in the ON position, it is 13.2 times better at limiting heat transfer!

Multi-Mode Heat Transfer (Radiation + Convection)

Assuming we have radiation and convection for example, our analysis is similar:

Our resistance diagram becomes: (T_{gas} is unknown)



We would have to solve the complete diagram, and it may require iteration, if T_1 or T_2 are not provided and instead q'' is provided.

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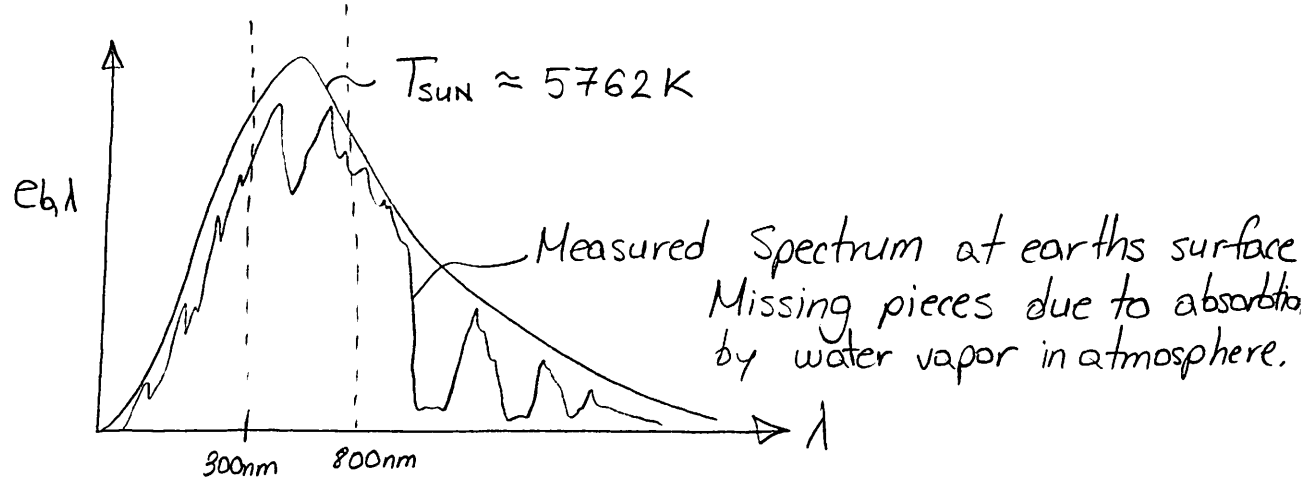
Solar Radiation



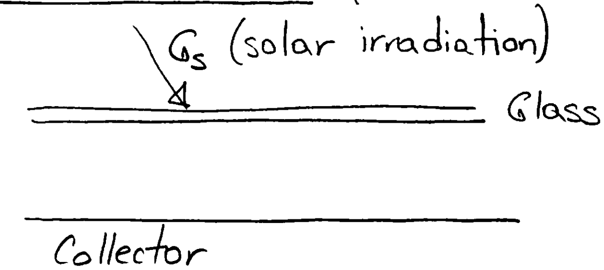
$$q_{sun} = 1353\ W/m^2 \text{ (Outside the atmosphere)}$$

$$q_{earth} = 636\ W/m^2 \text{ (On the earths surface)}$$

Note, the total arriving energy from the sun to the earth is $\approx 1.7 \times 10^{14}\,kW$. The US peak demand is $\approx 1 \times 10^{9}\,kW$.

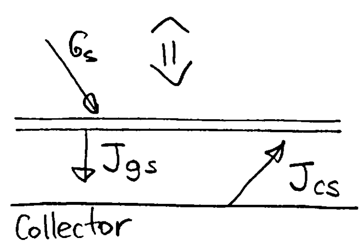
Our eyes are most sensitive to visible light (400-700nm) where the peak intensity from the sun is. This is simply due to evolution.

Solar Collectors: (Detailed Analysis)



glass : $\tau_{gs}, \rho_{gs}, \alpha_{gs}$

collector : ρ_{cs}, α_{cs}



Reflected energy J_{cs} from glass back to collector.

$$J_{gs} = G_s \tau_{gs} + \rho_{gs} J_{cs}$$

$$J_{cs} = \rho_{cs} J_{gs} \quad \text{\} Reflected energy from glass}$$

$$J_{gs} = G_s \tau_{gs} + \rho_{gs} \rho_{cs} J_{gs}$$

$$J_{gs} = \frac{G_s \tau_{gs}}{1 - \rho_{gs}\rho_{cs}}$$

$$\boxed{q_{collector,s} = \alpha_{cs} J_{gs} = G_s \frac{\tau_{gs}\alpha_{cs}}{1 - \rho_{gs}\rho_{cs}}} \Rightarrow \text{Heat Collected in the solar spectrum.}$$

So now if we examine the IR range:

$$q_{coll-glass} = \frac{e_{bc} - e_{bg}}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} \quad \text{(Two parallel plates)}$$

If we look at the glass heat input: (From the solar spectrum)

$$\underbrace{\alpha_{gs}}_{\text{solar absorbtivity}} \left(G_s + J_{cs}\right) = \alpha_{gs} G_s \left[1 + \frac{\rho_{cs}\tau_{gs}}{1 - \rho_{gs}\rho_{cs}}\right]$$

We also have heat input from the collector to the glass:

$$\boxed{\alpha_{gs} G_s \left[1 + \frac{\rho_{cs}\tau_{gs}}{1 - \rho_{gs}\rho_{cs}}\right] + \frac{e_{bc} - e_{bg}}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} = \varepsilon_g \left(e_{bg} - e_{b\infty}\right) + \bar{h}\left(T_g - T_\infty\right)}$$

↳ Energy balance at the glass.

Solving the above equations can give us our collector efficiency. Note here we had to split up the two spectral ranges since $\varepsilon = \text{constant} = f(\lambda)$

Current collectors use micro/nanotechnology to achieve high absorptivity in the solar spectrum ($300\,nm < \lambda < 2500\,nm$) and low thermal emissivity in the IR spectum ($\lambda > 2500\,nm$).

END OF LECTURE 27