3. **Concentric Cylinders**

\[ F_{12} = 1 \]
\[ \frac{A_1}{A_2} = \frac{r_1}{r_2} \]
\[ q_{1-2} = \frac{2\pi r_1 L (e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{r_1}{r_2} \left( \frac{1}{\varepsilon_2} - 1 \right)} \]

4. **Concentric Spheres**

\[ \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad ; \quad F_{12} = 1 \]
\[ q_{1-2} = \frac{4\pi r_1^2 (e_{b1} - e_{b2})}{\frac{1}{\varepsilon_1} + \frac{r_1^2}{r_2^2} \left( \frac{1}{\varepsilon_2} - 1 \right)} \]

**Radiation Shields**

They are often used to reduce heat losses

1. \( \varepsilon_1 \)
2. \( \varepsilon_2 \)

---

Drawing out our resistance diagram

\[ q_{12} = \frac{A (e_{b1} - e_{b2})}{\left( \frac{1}{\varepsilon_1} - 1 \right) + 1 + \left( \frac{1}{\varepsilon_2} - 1 \right) + 2 \left( \frac{1}{\varepsilon_{Sc}} - 1 \right) + 1} \]
\[
q_{12} = \frac{A \left( e_{b1} - e_{b2} \right)}{\left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right) + \frac{2}{\varepsilon_{sc}} - 1} \quad \Rightarrow \text{For 1 shield}
\]

For \( n \) shields in series:
\[
q_{12} = \frac{A \left( e_{b1} - e_{b2} \right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_{sc}} - (n+1)}
\]

For cylindrical shields (\( n \) of them):
\[
q_{12} = \frac{2\pi r_1 L \left( e_{b1} - e_{b2} \right)}{\frac{1}{\varepsilon_1} + \frac{r_1}{r_2} \left( \frac{1}{\varepsilon_2} - 1 \right) + 2 \sum_{j=1}^{n} \frac{r_1}{r_{sc,j}}}
\]

For spherical shields (\( n \) of them):
\[
q_{12} = \frac{4\pi r_1^2 \left( e_{b1} - e_{b2} \right)}{\frac{1}{\varepsilon_1} + \frac{r_1^2}{r_2^2} \left( \frac{1}{\varepsilon_2} - 1 \right) + 2 \sum_{j=1}^{n} \frac{r_1^2}{r_{sc,j}^2}}
\]

Note, for spherical & cylindrical shields, you want \( r_{sc} \) to be close to \( r_1 \) since you want the shield to see as little of itself as possible.

Looking back at our adiabatic example: \( (e_b = J \text{ at } R) \)
\[
\frac{1}{R_{\text{eff}}} = \frac{1}{A_1 F_{1R}} + \frac{1}{A_2 F_{2R}} + A_1 F_{12}
\]

\[
q_{12} = \frac{e_{b1} - e_{b2}}{A_1 \left( \frac{1}{\varepsilon_1} - 1 \right) + A_2 \left( \frac{1}{\varepsilon_2} - 1 \right) + R_{\text{eff}}}
\]

⇒ Note, our solution does not depend on \(E_R\) since \(e_{bR} = 0\) or \(q'' = 0\) at \(R\).

Example: Consider a rotatable radiation shield shown below. Calculate its effectiveness in the on & off positions:

\(\text{I: ON} \quad -e_{b\infty}\)

\(\text{II: OFF} \quad -e_{b\infty}\)

Solving the on case first:

\[
q_{1\infty} = \frac{A_1 (e_{b1} - e_{b\infty})}{\frac{1}{\varepsilon_1} + \frac{2}{\varepsilon_\infty} - 1} \quad F_{\infty,\infty} = 1
\]

Looking at the OFF case (surfaces are reflective since \(q'' = 0\) at the shields)

\[
q_{12} = \frac{A_1 (e_{b1} - e_{b\infty})}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}}
\]

\[
F_{12} = \sqrt{2} - 1 \quad F_{1R} = 2 - \sqrt{2}
\]

Aside: Note, the shields in \(\text{II}\) are adiabatic due to symmetry.
\( R_\| \gg R_\perp \), Assuming \( \varepsilon_1 = 0.5 \); \( R \) = thermal radiative resistance.

\[
\frac{R_\|}{R_\perp} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_\text{sc}} - 1}{\frac{1}{\varepsilon_1} - 1 + \sqrt{2}} = 13.2
\]

So when the shield is in the ON position, it is 13.2 times better at limiting heat transfer!

**Multi-Mode Heat Transfer (Radiation + Convection)**

Assuming we have radiation and convection for example, our analysis is similar:

\[ T_1 ~ h_1 ~ h_2 ~ T_{\text{gas}} ~ h_2 ~ T_2 \]

Our resistance diagram becomes: (\( T_{\text{gas}} \) is unknown)

\[
\sigma T_1^4 = e_{b1} \left( \frac{1 - \varepsilon_1}{A_1\varepsilon_1} \right) J_1 \quad \quad \quad J_2 \left( \frac{1 - \varepsilon_2}{A_2\varepsilon_2} \right) e_{b2} = \sigma T_2^4
\]

We would have to solve the complete diagram, and it may require iteration, if \( T_1 \) or \( T_2 \) are not provided and instead \( q \) is provided.
Solar Radiation

\[ T_{\text{sun}} \approx 5762 \text{K} \]

Measured Spectrum at earth's surface. Missing pieces due to absorption by water vapor in atmosphere.

\[ q_{\text{sun}} = 1353 \text{ W/m}^2 \text{ (Outside the atmosphere)} \]

\[ q_{\text{earth}} = 636 \text{ W/m}^2 \text{ (On the earth's surface)} \]

Note, the total arriving energy from the sun to the earth is \( \approx 1.7 \times 10^{14} \text{ kW} \). The US peak demand is \( \approx 1 \times 10^9 \text{ kW} \).

Our eyes are most sensitive to visible light (400-700nm) where the peak intensity from the sun is. This is simply due to evolution.

**Solar Collectors: (Detailed Analysis)**

\[ q_s \text{ (solar irradiation)} \]

\[ \downarrow \text{Glass} \]

\[ \downarrow \text{Collector} \]

Glass:
\[ q_{gs}, I_{gs}, \alpha_{gs} \]

Collector:
\[ q_{cs}, \alpha_{cs} \]

Reflected energy:
\[ J_{cs} = \alpha_{cs} q_{gs} \]

\[ J_{gs} = q_s - q_{gs} - q_{gs} \alpha_{cs} \]

\[ J_{gs} = q_s - q_{gs} - \alpha_{cs} q_{gs} \]

\[ J_{gs} = q_s - q_{gs} (1 - \alpha_{cs}) \]
\[ J_{gs} = \frac{C_s C_{gs}}{1 - P_{gs} P_{cs}} \]

\[ Q_{\text{collector}} = \alpha_c s J_{gs} = C_s \frac{C_{gs} \alpha_c s}{1 - P_{gs} P_{cs}} \Rightarrow \text{Heat Collected in the solar spectrum.} \]

So now if we examine the IR range:

\[ Q_{\text{coll-glass}} = \frac{E_{bc} - E_{bg}}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} \quad \text{(Two parallel plates)} \]

If we look at the glass heat input: (From the solar spectrum)

\[ \alpha_{gs} (C_s + J_{cs}) = \alpha_{gs} C_s \left[ 1 + \frac{P_{cs} C_{gs}}{1 - P_{gs} P_{cs}} \right] \]

\[ \alpha_{gs} C_s \left[ 1 + \frac{P_{cs} C_{gs}}{1 - P_{gs} P_{cs}} \right] + \frac{E_{bc} - E_{bg}}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_g} - 1} = \varepsilon_g (E_{bg} - E_{\infty}) + \theta (T_g - T_{\infty}) \]

\[ \Rightarrow \text{Energy balance at the glass.} \]

Solving the above equations can give us our collector efficiency. Note here we had to split up the two spectral ranges since \( \varepsilon = \text{constant} = f(\lambda) \)

Current collectors use micro/nanotechnology to achieve high absorptivity in the solar spectrum (300nm ≤ \( \lambda \) ≤ 2500nm) and low thermal emissivity in the IR spectrum (\( \lambda > 2500 \) nm).