Convective Mass Transfer (External)

If we examine air blowing over a flat plate

We've already solved this problem:

\[ n = \frac{y}{\sqrt{\frac{x}{V_{\infty}}}} \]

Now if we look at the mass transfer analogy (N-S eqn.)

\[ U \frac{\partial C_i}{\partial x} + V \frac{\partial C_i}{\partial y} = D_{i,g} \frac{\partial^2 C_i}{\partial y^2} \]

Note, this is the exact same equation as in heat transfer, so we can make a really simple analogy and our mass transfer results will be the same.
\[
\frac{C_i - C_{i,\infty}}{C_{i,0} - C_{i,\infty}} \quad \text{or} \quad \frac{\Delta T/T_\infty}{T_\infty - T_0}
\]

\[
z = \frac{y}{\sqrt{U x / V_\infty}}, \quad n^* = n Pr^{1/3}
\]

For the heat transfer case: \( Pr = \frac{V}{\alpha} \)

For the mass transfer case: \( Sc = \frac{V}{D_{i,2}} \)

Schmidt number

\( \alpha \) = kinematic viscosity

\( D_{i,2} \) = diffusivity (mass)

Our solutions become:

\[
\text{Nu} = \frac{h x}{k} = 0.332 \text{Re}_x^{1/2} Pr^{1/3}
\]

\[
\text{Nu}_L = \frac{h L}{k} = 0.664 \text{Re}_L^{1/2} Pr^{1/3}
\]

For mass transfer:

\[
\text{Sh} = \frac{h_m x}{D} = 0.332 \text{Re}_x^{1/2} Sc^{1/3}
\]

Sherwood number

We can now develop a Colburn analogy for mass transfer to express the mass transfer coefficient in terms of friction factor, \( f \): (pg. 90 of notes)

\[
C_f = \frac{\Delta P}{\frac{1}{2} \rho V_\infty^2} = \frac{2A_2}{\sqrt{\text{Re}_x}}, \quad A_2 = 0.332
\]

\[
\text{Nu}_x = A_2 \text{Re}_x^{1/2} Pr^{1/3} \quad \text{(Flat plate)}
\]
\[
\left( \frac{Nu}{Pr} \right) = \frac{1}{2} \frac{2 \alpha}{\sqrt{Re_x \cdot Pr^{2/3}}} \cdot \frac{1}{Pr^{2/3}} = \frac{1}{2} \frac{C_f}{Pr} \frac{1}{Pr^{2/3}}
\]

\[
St = \frac{h}{\rho C_p V_\infty} \quad \text{(Santon number)} = \frac{\text{heat transferred to a fluid}}{\text{thermal capacity of the fluid}}
\]

So our heat transfer analogy was:

\[
St \cdot Pr^{2/3} = \frac{C_f}{2} \Rightarrow \text{See page 90 of notes for more.}
\]

For mass transfer, we have:

\[
\left( \frac{Sh}{Re_x \cdot Sc} \right) = \frac{1}{2} \frac{2 \alpha}{\sqrt{Re_x} \cdot Sc^{2/3}} \cdot \frac{1}{Sc^{2/3}} = \frac{1}{2} \frac{C_f}{Sc} \frac{1}{Sc^{2/3}} = St_m
\]

By analogy (you can prove it):

\[
St_m \cdot Sc^{2/3} = \frac{C_f}{2}
\]

Mass transfer Colburn analogy

\[
St_m = \frac{h_m}{V_\infty}
\]

Mass transfer Stanton number

Notice that we can combine the two analogies:

\[
St \cdot Pr^{2/3} = St_m \cdot Sc^{2/3}
\]

\[
\frac{h}{\rho C_p V_\infty} \cdot Pr^{2/3} = \frac{h_m}{V_\infty} \cdot Sc^{2/3}
\]

\[
\frac{h}{h_m} = \left( \rho C_p \right) \cdot \left( \frac{\alpha}{D_{1,2}} \right)^{2/3} = \rho \cdot C_p \cdot Le^{2/3}, \quad \text{where} \quad Le = \frac{\alpha}{D_{1,2}}
\]

\[
eq \text{Lewis number}
\]

Note this means we can measure mass transfer \((h_m)\) and use it to back calculate heat transfer \((h)\) \(\Rightarrow\) very powerful
Example: Dry air at atmospheric pressure blows across a thermometer which is enclosed in a damp cover. This is a classical wet bulb measurement. The temperature reads 18.3°C. What is the dry air temperature, \( T_{\infty} \)?

The heat needed to evaporate the water comes from the surrounding air (at steady state):

\[
hA (T_{\infty} - T_w) = m_w h_{fg} \quad (1) \Rightarrow \text{Heat transfer}
\]

\[
m_w = h_m A (\rho_w - \rho_{\infty}) \quad (2) \Rightarrow \text{Mass transfer}
\]

Combining (1) and (2)

\[
hA (T_{\infty} - T_w) = h_m A (\rho_w - \rho_{\infty}) h_{fg}
\]

Using our Calburn analogies

\[
\rho_w C_p \text{Le}^{2/8} (T_{\infty} - T_w) = (\rho_{\infty} - \rho_{\infty}) h_{fg} \quad (3)
\]

We know \( C_w \) corresponds to the saturation conditions at \( T_w \). From steam tables:

\[
P_{\text{sat}} = 2107 \text{ Pa at } T_{\text{sat}} = 18.3^\circ \text{C}
\]

\[
\rho_w = \frac{P_{\text{sat}}}{R_w T_w} = \frac{(2107)(18)}{(8315)(291.3)} = 0.01566 \text{ kg/m}^3
\]

For the other properties:

\[
\rho_{\infty} = 0 \text{ (dry air)}
\]

\[
\rho_a = \text{air density} = \frac{\rho}{RT} = \frac{101325}{(287)(291.3)} = 1.212 \text{ kg/m}^3
\]
\( c_p = 1.004 \text{ kJ/kg} \)

\( \frac{\alpha}{D_{1/2}} = \text{Le} = \frac{Sc}{Pr} = 0.845 \)

\( h_{fg} = 2.456 \text{ MJ/kg} \)

Back substituting into equation (3)

\[
T_\infty - T_w = \frac{(0.01566 - 0)(2.456 \times 10^6)}{(1.212)(1004)(0.845)^{2/3}} = 35.36^\circ C
\]

\[
T_\infty = 53.69^\circ C \implies \text{Dry bulb temperature}
\]

**Mass Transfer Summary**

<table>
<thead>
<tr>
<th>Heat Transfer</th>
<th>Mass Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h ) ( [W/m^2 \cdot K] )</td>
<td>( h_m ) ( [m/s] )</td>
</tr>
<tr>
<td>( Re ) ( [\text{dimensionless}] )</td>
<td>( Re )</td>
</tr>
<tr>
<td>( Nu = \frac{hL}{k_f} = \text{Nusselt} )</td>
<td>( Sh = \frac{h_mL}{D_{1/2}} = \text{Sherwood} )</td>
</tr>
<tr>
<td>( Nu = f(Re, Pr) )</td>
<td>( Sh = f(Re, Sc) )</td>
</tr>
<tr>
<td>( Pr = \frac{v}{\alpha} = \text{Prandtl} )</td>
<td>( Sc = \frac{v}{D_{1/2}} = \text{Schmidt} )</td>
</tr>
<tr>
<td>( Ra = \frac{g \beta \Delta T L^3}{\alpha \alpha} = \text{Rayleigh} )</td>
<td>( Ra_m = \frac{g \Delta \rho L^3}{\mu D_{1/2}} = \text{Rayleigh mass transf. (Natural conv.)} )</td>
</tr>
<tr>
<td>( Le = \frac{\alpha}{D_{1/2}} )</td>
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