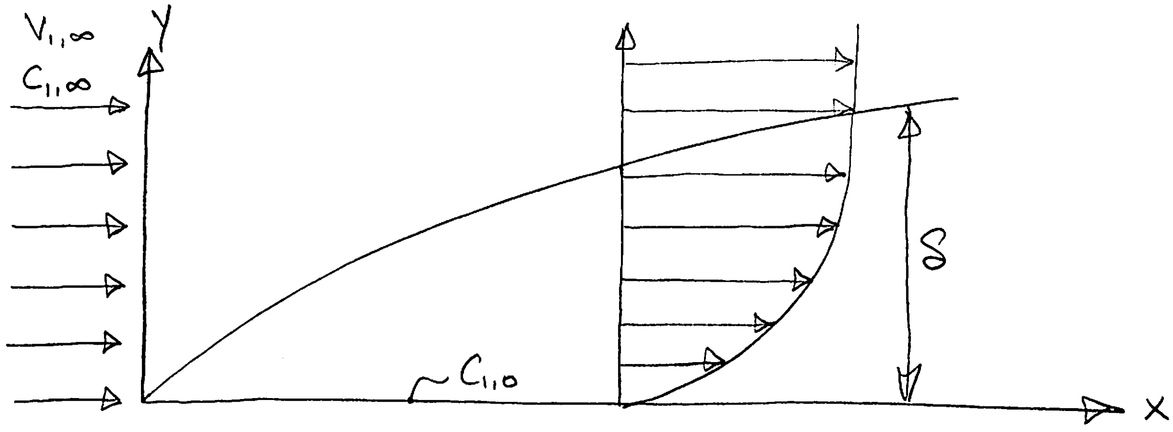
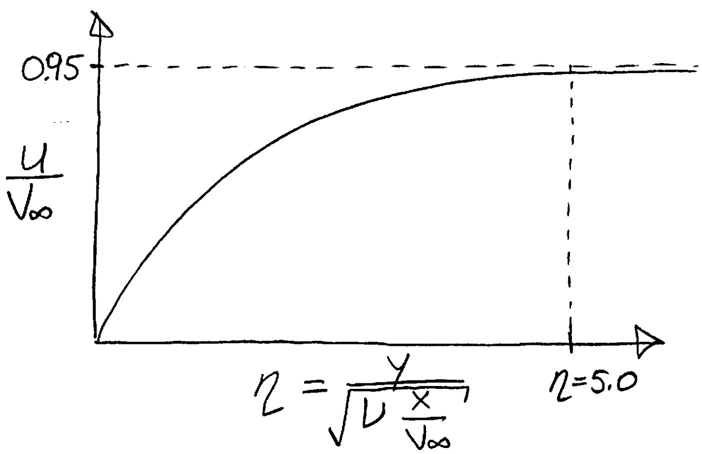
Convective Mass Transfer (External)

If we examine air blowing over a flat plate



We've already solved this probem:



⇒ See page 84 of notes.

Now if we look at the mass transfer analogy (N-S eqn.)

$$u\frac{\partial C_1}{\partial x} + v\frac{\partial C_1}{\partial y} = D_{1,g}\frac{\partial^2 C_1}{\partial y^2}$$

Note, this is the exact same equation as in heat transfer, so we can make a really simple analogy and our mass transfer results will be the same

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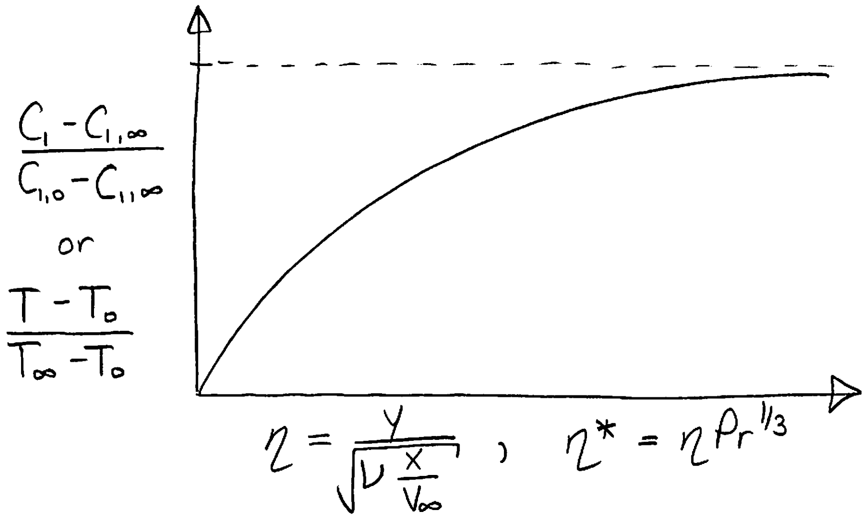

⇒ See page 89 of notes

For the heat transfer case: $Pr = \frac{\nu}{\alpha}$

For the mass transfer case: $\boxed{Sc = \frac{\nu}{D_{1,2}}}$ Schmidt number

ν = kinematic viscosity

$D_{1,2}$ = diffusivity (mass)

Our solutions become:

$$Nu = \frac{hx}{k} = 0.332\, Re_x^{1/2} Pr^{1/3}$$

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = 0.664\, Re_L^{1/2} Pr^{1/3}$$

For mass transfer:

$$\boxed{Sh = \frac{h_m x}{D} = 0.332\, Re_x^{1/2} Sc^{1/3}}$$ Sherwood number

We can now develop a Colburn analogy for mass transfer to express the mass transfer coefficient in terms of friction factor, f: (pg. 90 of notes)

$$C_{f,x} = \frac{\tau}{\frac{1}{2}\rho V_\infty^2} = \frac{2a_2}{\sqrt{Re_x}}, \quad a_2 = 0.332$$

$$Nu_x = a_2 Re_x^{1/2} Pr^{1/3} \quad \text{(Flat plate)}$$

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$$\underbrace{\left(\frac{Nu_x}{Re_x \cdot Pr}\right)}_{} = \frac{1}{2}\,\frac{2a_2}{\sqrt{Re_x}}\,\frac{1}{Pr^{2/3}} = \frac{1}{2}\,C_{f,x}\,\frac{1}{Pr^{2/3}}$$

$$St = \frac{h}{\rho C_p V_\infty} \quad \text{(Santon number)} \equiv \frac{\text{heat transfered to a fluid}}{\text{thermal capacity of the fluid}}$$

So our heat transfer analogy was:

$$St \cdot Pr^{2/3} = \frac{C_{f,x}}{2}$$

⇒ See page 90 of notes for more.

For mass transfer, we have:

$$\left(\frac{Sh_x}{Re_x\, Sc}\right) = \frac{1}{2}\,\frac{2a_2}{\sqrt{Re_x}} \cdot \frac{1}{Sc^{2/3}} = \frac{1}{2}\,C_{f,x}\,\frac{1}{Sc^{2/3}} = St_m$$

By analogy (you can proove it)

$$\boxed{St_m \cdot Sc^{2/3} = \frac{C_{f,x}}{2}} \quad \text{Mass transfer Colburn analogy}$$

$$\boxed{St_m = \frac{h_m}{V_\infty}} \quad \text{Mass transfer Stanton number}$$

Notice that we can combine the two analogies

$$St \cdot Pr^{2/3} = St_m \cdot Sc^{2/3}$$

$$\frac{h}{\rho C_p V_\infty} Pr^{2/3} = \frac{h_m}{V_\infty} Sc^{2/3}$$

$$\boxed{\frac{h}{h_m} = (\rho C_p)\left(\frac{\alpha}{D_{1,2}}\right)^{2/3} = \rho \cdot C_p \cdot Le^{2/3}}, \quad \text{where } \boxed{Le = \frac{\alpha}{D_{1,2}}}$$

$\equiv$ Lewis number

Note this means we can measure mass transfer (h_m) and use it to back calculate heat transfer (h) ⇒ very powerful

<u>Example</u> | Dry air at atmospheric pressure blows across a thermometer which is enclosed in a damp cover. This is a classical wet bulb measurement. The temperature reads 18.3°C. What is the dry air temperature, T_∞?

The heat needed to evaporate the water comes from the surrounding air (at steady state).

$$hA(T_\infty - T_w) = \dot{m}_w h_{fg} \quad ① \Rightarrow \text{Heat transfer}$$

$$\dot{m}_w = h_m A(\rho_w - \rho_\infty) \quad ② \Rightarrow \text{Mass transfer}$$

Combining ① and ②

Mass concentrations

$$h\cancel{A}(T_\infty - T_w) = h_m \cancel{A}(\rho_w - \rho_\infty)h_{fg}$$

Using our Colburn analogies

$$\rho_a C_p Le^{2/3}(T_\infty - T_w) = (\rho_w - \rho_\infty)h_{fg} \quad ③$$

We know C_w corresponds to the saturation conditions at T_w. From steam tables:

$$P_{sat} = 2107 \text{ Pa at } T_{sat} = 18.3°C$$

$$\rho_w = \frac{P_{sat}}{R_w T_w} = \frac{(2107)(18)}{(8315)(291.3)} = 0.01566 \text{ kg/m}^3$$

For the other properties:

$\rho_\infty = 0$ (dry air)

$$\rho_a = \text{air density} = \frac{P}{RT} = \frac{101325}{(287)(291.3)} = 1.212 \text{ kg/m}^3$$

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$$C_p = 1.004\ kJ/kg$$

$$\frac{\alpha}{D_{1,2}} = Le = \frac{Sc}{Pr} = 0.845$$

$$h_{fg} = 2.456\ MJ/kg$$

Back substituting into equation ③

$$T_\infty - T_w = \frac{(0.01566 - 0)(2.456 \times 10^6)}{(1.212)(1004)(0.845)^{2/3}} = 35.36^\circ C$$

$$\therefore \boxed{T_\infty = 53.69^\circ C} \Rightarrow \text{Dry bulb temperature}$$

Mass Transfer Summary

Heat Transfer	Mass Transfer
$h\ [W/m^2 \cdot K]$	$h_m\ [m/s]$
Re [dimensionless]	Re
$Nu = \frac{hL}{k_f} \equiv$ Nusselt	$Sh = \frac{h_m L}{D_{1,2}} \equiv$ Sherwood
$Nu = f(Re, Pr)$	$Sh = f(Re, Sc)$
$Pr = \frac{\nu}{\alpha} \equiv$ Prandtl	$Sc = \frac{\nu}{D_{1,2}} \equiv$ Schmidt
$Ra = \frac{g\beta\Delta T L^3}{\nu\alpha} \equiv$ Rayleigh	$Ra_m = \frac{g\Delta\rho L^3}{\mu D_{1,2}} \equiv$ Rayleigh mass transf. (Natural conv.)
	$Le = \frac{\alpha}{D_{1,2}}$