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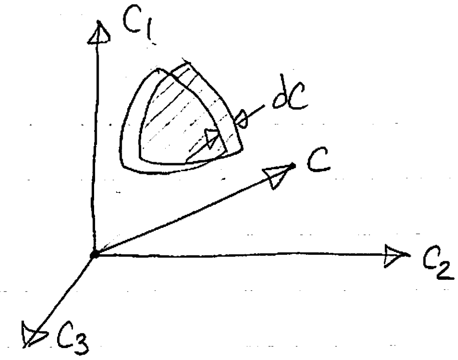
$$\left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{m(c_1^2+c_2^2+c_3^2)}{2k_BT}}\, dc_1\, dc_2\, dc_3 = 1$$

we know that $c^2 = c_1^2 + c_2^2 + c_3^2$

$$\left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{mc^2}{2k_BT}}\, dc_1\, dc_2\, dc_3$$




For a sphere: $4\pi c^2$ (surface area)

$$dV_c = \underbrace{4\pi c^2 dc}_{\text{Thin shell approx.}} = dc_1 dc_2 dc_3$$

Our integral becomes much simpler.

$$4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^{\infty} e^{-\frac{mc^2}{2k_BT}} c^2 dc = 1$$

So we previously defined $\hat{c} = c\left(\frac{m}{2k_BT}\right)^{1/2} \Rightarrow c = \frac{\hat{c}}{\left(\frac{m}{2k_BT}\right)^{1/2}}$

$$d\hat{c} = \left(\frac{m}{2k_BT}\right)^{1/2} dc$$

Back substituting:

$$= \frac{4\pi}{\pi^{3/2}} \left(\frac{m}{\cancel{2k_BT}}\right)^{\cancel{3/2}} \int_0^{\infty} e^{-\hat{c}^2} \hat{c}^2 \left(\frac{1}{\left(\frac{m}{\cancel{2k_BT}}\right)}\right) \frac{d\hat{c}}{\left(\frac{m}{\cancel{2k_BT}}\right)^{\cancel{1/2}}}$$

$$= \frac{4}{\sqrt{\pi}} \underbrace{\int_0^{\infty} \hat{c}^2 e^{-\hat{c}^2} d\hat{c}}_{I_2 = \frac{\sqrt{\pi}}{4}} = 1 \Rightarrow \text{Works out}$$

From this, we see that the general speed distribution is:

$$\boxed{\Psi(c) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_BT}\right)^{3/2} c^2 e^{-\frac{mc^2}{2k_BT}}}$$

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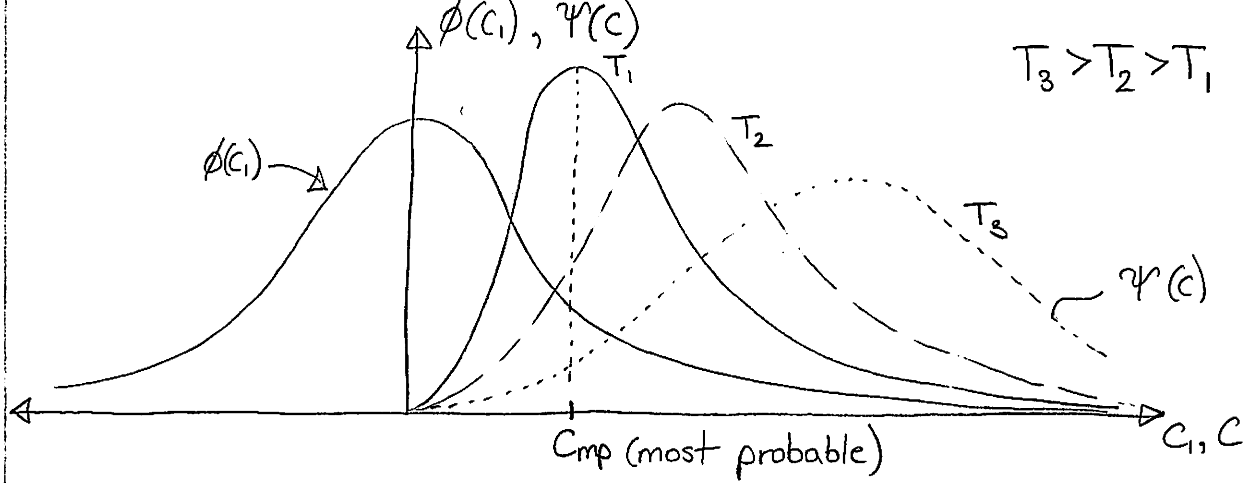
So what do our distributions look like?




So if we want to calculate the mean molecule speed $(\bar{c})$

$$\bar{c} = \int_0^{\infty} c\,\Psi(c)\,dc$$

$$\bar{c} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}\int_0^{\infty} c^3 e^{-\frac{mc^2}{2k_BT}}\,dc$$

Doing our usual tricks $\hat{c} = c\left(\frac{m}{2k_BT}\right)^{1/2}$

$$\bar{c} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}\left(\frac{2k_BT}{m}\right)^{2}\underbrace{\int_0^{\infty}\hat{c}^3 e^{-\hat{c}^2}\,d\hat{c}}_{\text{We know this from our previous calculations as } I_3 = \frac{1}{2}}$$

Aside:

$$\int_0^{\infty}\hat{c}^3 e^{-\hat{c}^2}\,d\hat{c} = I_3$$

$$I_3 = \frac{n-1}{2} I_{n-2} = \frac{3-1}{2} I_1 = \frac{2}{2}\left(\frac{1}{2}\right)$$

$$\bar{c} = \frac{1}{2}\,\frac{\cancel{4}\,2}{\sqrt{\pi}}\left(\frac{\cancel{m}}{\cancel{2k_BT}}\right)^{3/2}\left(\frac{2k_BT}{m}\right)^{\cancel{2}\ \frac{1}{2}}$$

$$\boxed{\bar{c} = \sqrt{\frac{8k_BT}{\pi m}}}$$ $\Rightarrow$ Average speed $\Rightarrow$ $\bar{c} \sim \sqrt{T}$, as $T\uparrow, \bar{c}\uparrow$

So how about $\overline{c^2}$?

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$$\overline{c^2} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}\int_0^\infty c^4 e^{-\frac{mc^2}{2k_BT}}\,dc \Rightarrow \hat{c} = c\left(\frac{m}{2k_BT}\right)^{1/2}$$

$$\overline{c^2} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}\left(\frac{2k_BT}{m}\right)^{5/2}\underbrace{\int \hat{c}^4 e^{-\hat{c}^2}\,d\hat{c}}_{I_4}$$

$$I_4 = \frac{n-1}{2}I_{n-2} = \frac{4-1}{2}I_{4-2} = \frac{3}{2}I_2 = \frac{3}{2}\frac{\sqrt{\pi}}{4} = \frac{3\sqrt{\pi}}{8}$$

$$\overline{c^2} = \frac{\cancel{4}}{\cancel{\sqrt{\pi}}}\cdot\frac{3\cancel{\sqrt{\pi}}}{\cancel{8}\,2}\cancel{\left(\frac{m}{2k_BT}\right)^{3/2}}\left(\frac{2k_BT}{m}\right)^{\cancel{5/2}\,1}$$

$$= \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}k_BT}{m}$$

$$\boxed{\overline{c^2} = \frac{3k_BT}{m}} \quad \text{or} \quad \boxed{c_{rms} = \sqrt{\frac{3kT}{m}}} \Rightarrow \text{Root mean square speed.}$$

Looks familiar right?
We've prooven that our average molecule kinetic energy assumption is indeed correct:

$$\frac{1}{2}m\overline{c^2} \approx \frac{3}{2}k_BT$$

How about the most probable velocity?
For this, we need to differentiate: $\frac{d}{dc}(\Psi(c)) = 0$

$$\frac{d}{dc}\left(\frac{4}{\sqrt{\pi}}\cdot\left(\frac{m}{2k_BT}\right)^{3/2}c^2 e^{-\frac{mc^2}{2k_BT}}\right) = 0$$

$$\cancel{\frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}}\frac{d}{dc}\left(c^2 e^{-\frac{mc^2}{2k_BT}}\right) = 0$$

$$\frac{d}{dc}(c^2)\cdot e^{-\frac{mc^2}{2k_BT}} + c^2\frac{d}{dc}\left(e^{-\frac{mc^2}{2k_BT}}\right) = 0$$

$$\cancel{2c}\,\cancel{e^{-\frac{mc^2}{2k_BT}}} + c^2\left(-\frac{\cancel{2}m\cancel{c}}{2k_BT}\right)\cancel{e^{-\frac{mc^2}{2k_BT}}} = 0$$

$$1 - \frac{mc^2}{2k_BT} = 0 \Rightarrow \boxed{c_{mp} = \sqrt{\frac{2k_BT}{m}}} \Rightarrow \text{most probable speed.}$$

To give you some insight, the speed of sound is $c_{sound} \approx 0.84\,c_{mp}$ and $c_{sound} \approx 0.74\,c_{mp}$ ⇒ Since sound waves propagate via molecular motion, makes sense that they are slighty slower than $c_{mp}$.

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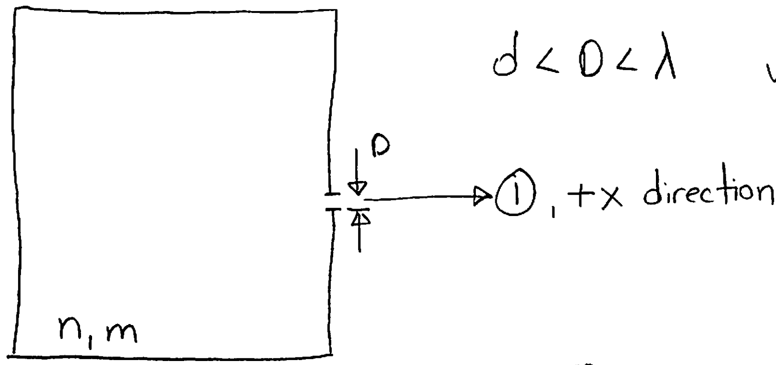
**Example #2** Reservoir full of air. What is the mass flux out of the orifice?




$d < D < \lambda$ where
$n$ = # of molecules/volume
$d$ = size of molecule
$D$ = orifice opening
$\lambda$ = mean free path
$m$ = mass per molecule
$A$ = orifice area

$$\dot{m} = nmA\int_0^\infty c_1\,\phi(c_1)\underbrace{\int_{-\infty}^{\infty}\phi(c_2)\,dc_2}_{1}\ \underbrace{\int_{-\infty}^{\infty}\phi(c_3)\,dc_3}_{1}$$

$\Rightarrow$ We don't care about the molecules y or z velocity

Note, 0 is lower bound since we only want molecules that travel in the +x direction out of the reservoir.

$$\dot{m} = nmA\frac{1}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{1/2}\int_0^\infty c_1\,e^{-\frac{mc_1^2}{2k_BT}}\,dc_1$$

Using our old trick, and knowing that $n = \frac{\rho}{m}$

$$\dot{m} = \rho A\frac{1}{\sqrt{\pi}}\left(\frac{2k_BT}{m}\right)^{1/2}\underbrace{\int_0^\infty \hat{c}_1\,e^{-\hat{c}_1^{\,2}}\,d\hat{c}_1}_{I_1 = \frac{1}{2}}$$

$$= \frac{\rho A}{2\sqrt{\pi}}\left(\frac{2k_BT}{m}\right)^{1/2}$$

$$= \frac{\rho A}{2}\left(\frac{2k_BT}{\pi m}\right)^{1/2} \Rightarrow \text{multiply by } \left(\frac{2}{2}\right)$$

$$= \frac{\rho A}{4}\underbrace{\left(\frac{8k_BT}{\pi m}\right)^{1/2}}_{\bar{c}} = \boxed{\frac{\rho A\bar{c}}{4} = \dot{m}}$$

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Another way to write it is:

$$\dot{m} = \frac{pA}{4}\left(\frac{8k_BT}{\pi m}\right)^{1/2} = \frac{\sqrt{8}\,pA}{4\sqrt{\pi}}\sqrt{\frac{k_BT}{m}} = \frac{2^{3/2}\,pA}{2^2\pi^{1/2}}\sqrt{\frac{k_BT}{m}}$$

We also know that: $R_{gas} = \frac{k_B}{m}$ Aside $\Rightarrow$

$$R_{gas} = \frac{R}{M}$$

$$k_B = \frac{R}{A} = \frac{MR_{gas}}{A} \leftarrow \text{Avogadro NOT! Area.}$$

but $M = A\cdot m$

$$k_B = \frac{A\,m\,R_{gas}}{A} = R_{gas}\,m$$

$$\boxed{\dot{m} = \frac{pA}{\sqrt{2\pi}}\sqrt{R_{gas}T} \approx 0.4\,pA\sqrt{RT}}$$

For comparison, what if $D > \lambda$, so we can use continuum mechanics to solve for $\dot{m}$. Now we have a choked flow condition. From compressible fluid mechanics:

$$\dot{m} = pA\sqrt{R_{gas}T}\cdot\underbrace{\left[\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{1/2}}_{0.684}$$

$$\boxed{\dot{m} = 0.684\,pA\sqrt{RT}} \Rightarrow \text{Using continuum mechanics}$$

We overestimate the result by 50% using continuum mechanics when compared to rarified gas theory.

So what is the energy of the escaping gas molecules? Solve for $\overline{c^2}$ and $c_{esc}$. You will see that

$$\overline{c^2_{esc}} = 4\,\frac{R_{gas}T}{m} = \frac{4k_BT}{m}$$

$$\boxed{\frac{4k_BT}{m} > \frac{3k_BT}{m} = \langle E\rangle_{Reservoir}}$$

↳ Escaping molecules have on average a larger energy than the rest of the molecules in the reservoir. Makes sence, they travel faster and scatter off the walls more frequently.

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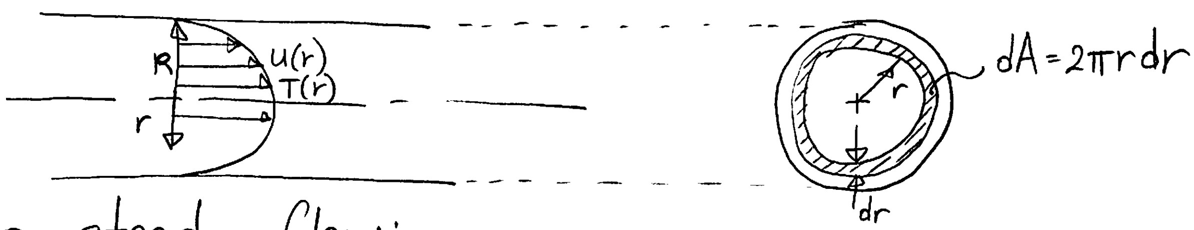
## Extra Derrivation (Pinhole problem)

Solution to energy of escaping gas molecules from a pinhole:

The way to solve this problem is analogous to calculating bulk flow temperature of a fluid in a pipe.




For a steady flow:

$$\bar{u} = \frac{1}{\rho \pi R^2}\int_0^R \rho u(r) 2\pi r\, dr$$

$$\bar{T} = \frac{1}{\rho c_p \bar{u} \pi R^2}\int_0^R \underbrace{\rho c_p T(r) u(r)\, 2\pi r\, dr}_{\text{Total convected energy in the flow.}}$$

So we now have:

$$\bar{u} = \frac{1}{\pi R^2}\int_0^R u\, 2\pi r\, dr$$

$$\boxed{\bar{T} = \frac{1}{\pi R^2 \bar{u}}\int_0^R (uT)\, 2\pi r\, dr}$$

⇒ We will need to use an anologous approach.

In our case, we need $\overline{C_{esc}^2}$. We can write:

$$\overline{C_{esc}^2} = \frac{1}{\underbrace{\overline{C_{esc}}}}\left(\frac{m}{2k_B T}\right)^{3/2}\frac{1}{\pi^{3/2}}\underbrace{\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty C_1\, (C^2)\, e^{-\frac{mc^2}{2k_B T}}\, dC_1\, dC_2\, dC_3}$$

Need to divide by $\overline{C_{esc}}$ due to the $C_1$ in the integral.

$(C^2)$ → energy advected

$C_1$ → in the 1th component

This term is like the $\bar{T}$ calculation. We are computing the average energy, $\overline{C^2}$, advected in the $C_1$ direction

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We can aslo express $\overline{C_{esc}}$ as: $\overline{C_{esc}} = \frac{\dot{m}}{\rho A}$

Expanding our integral and back substituting $\overline{C_{esc}}$:

$$\overline{C_{esc}^{2}} = \frac{\rho A}{\dot{m}}\left(\frac{m}{2k_BT}\right)^{3/2}\frac{1}{\pi^{3/2}}\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty C_1\left(C_1^2+C_2^2+C_3^2\right)e^{-\frac{mC_1^2}{2k_BT}}e^{-\frac{mC_2^2}{2k_BT}}e^{-\frac{mC_3^2}{2k_BT}}\cdot dC_1\,dC_2\,dC_3$$

Now we can expand the integral

$$\overline{C_{esc}^{2}} = \underbrace{\frac{\rho A}{\dot{m}}\left(\frac{m}{2k_BT}\right)^{3/2}\frac{1}{\pi^{3/2}}}_{X}\Bigg[\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty C_1^3 e^{-\frac{mC_1^2}{2k_BT}}e^{-\frac{mC_2^2}{2k_BT}}e^{-\frac{mC_3^2}{2k_BT}}\,dC_1\,dC_2\,dC_3$$

$$+\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty C_1 C_2^2 e^{-\frac{mC_1^2}{2k_BT}}e^{-\frac{mC_2^2}{2k_BT}}e^{-\frac{mC_3^2}{2k_BT}}\,dC_1\,dC_2\,dC_3$$

$$+\int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty C_1 C_3^2 e^{-\frac{mC_1^2}{2k_BT}}e^{-\frac{mC_2^2}{2k_BT}}e^{-\frac{mC_3^2}{2k_BT}}\,dC_1\,dC_2\,dC_3\Bigg]$$

$$\overline{C_{esc}^{2}} = X\Bigg[\int_0^\infty C_1^3 e^{-\frac{mC_1^2}{2k_BT}}dC_1\int_{-\infty}^\infty e^{-\frac{mC_2^2}{2k_BT}}dC_2\int_{-\infty}^\infty e^{-\frac{mC_3^2}{2k_BT}}dC_3$$

$$+\int_0^\infty C_1 e^{-\frac{mC_1^2}{2k_BT}}dC_1\int_{-\infty}^\infty C_2^2 e^{-\frac{mC_2^2}{2k_BT}}dC_2\int_{-\infty}^\infty e^{-\frac{mC_3^2}{2k_BT}}dC_3$$

$$+\int_0^\infty C_1 e^{-\frac{mC_1^2}{2k_BT}}dC_1\int_{-\infty}^\infty e^{\frac{mC_2^2}{2k_BT}}dC_2\int_{-\infty}^\infty C_3^2 e^{-\frac{mC_3^2}{2k_BT}}dC_3\Bigg]$$

Now we can normalize to make our integral simpler

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Let: $c_1 = \dfrac{\hat{c}_1}{\left(\frac{m}{2k_BT}\right)^{1/2}}$ , $c_2 = \dfrac{\hat{c}_2}{\left(\frac{m}{2k_BT}\right)^{1/2}}$ , $c_3 = \dfrac{\hat{c}_3}{\left(\frac{m}{2k_BT}\right)^{1/2}}$

$dc_1 = \dfrac{d\hat{c}_1}{\left(\frac{m}{2k_BT}\right)^{1/2}}$ , $dc_2 = \dfrac{d\hat{c}_2}{\left(\frac{m}{2k_BT}\right)^{1/2}}$ , $dc_3 = \dfrac{d\hat{c}_3}{\left(\frac{m}{2k_BT}\right)^{1/2}}$

Back substituting

$$\overline{c_{esc}^{\,2}} = X \underbrace{\left(\frac{2k_BT}{m}\right)^{3/2}}_{\text{from } \hat{c}} \underbrace{\left(\frac{2k_BT}{m}\right)^{3/2}}_{\text{from } d\hat{c}} \cdot \Bigg[ \underbrace{\int_0^\infty \hat{c}_1 e^{-\hat{c}_1^2} d\hat{c}_1}_{I_3} \underbrace{\int_{-\infty}^\infty e^{-\hat{c}_2^2} d\hat{c}_2}_{2I_0} \underbrace{\int_{-\infty}^\infty e^{-\hat{c}_3^2} d\hat{c}_3}_{2I_0}$$

$$+ \underbrace{\int_0^\infty \hat{c}_1 e^{\hat{c}_1^2} d\hat{c}_1}_{I_1} \underbrace{\int_{-\infty}^\infty \hat{c}_2 e^{-\hat{c}_2^2} d\hat{c}_2}_{2I_2} \underbrace{\int_{-\infty}^\infty e^{-\hat{c}_3^2} d\hat{c}_3}_{2I_0}$$

$$+ \underbrace{\int_0^\infty \hat{c}_1 e^{-\hat{c}_1^2} d\hat{c}_1}_{I_1} \underbrace{\int_{-\infty}^\infty e^{-\hat{c}_2^2} d\hat{c}_2}_{2I_0} \underbrace{\int_{-\infty}^\infty \hat{c}_3^2 e^{-\hat{c}_3^2} d\hat{c}_3}_{2I_2}$$

Putting everything together,

$$\overline{c_{esc}^{\,2}} = \frac{pA}{\dot{m}} \cancel{\left(\frac{m}{2k_BT}\right)^{3/2}} \frac{1}{\pi^{3/2}} \left(\frac{2k_BT}{m}\right)^{3/2} \left[ I_3 (2I_0)^2 + I_1(2I_2)(2I_0) + 4I_0I_1I_2 \right]$$

$$= \frac{pA}{\dot{m}} \frac{1}{\pi^{3/2}} \left(\frac{2k_BT}{m}\right)^{3/2} \left[ \frac{1}{2}\left(2\frac{\sqrt{\pi}}{2}\right)^2 + \frac{1}{2}\left(2\frac{\sqrt{\pi}}{4}\right)\left(2\frac{\sqrt{\pi}}{2}\right) + \frac{1}{2}\left(2\frac{\sqrt{\pi}}{2}\right)\left(2\frac{\sqrt{\pi}}{4}\right) \right.$$

$$= \frac{pA}{\dot{m}} \frac{1}{\pi^{3/2}} \left(\frac{2k_BT}{m}\right)^{3/2} \left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{pA}{\dot{m}} \left(\frac{2k_BT}{m}\right)^{3/2} \frac{1}{\sqrt{\pi}}$$

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But we already solved that

$$\dot{m} = \frac{pA}{4}\left(\frac{8k_BT}{\pi m}\right)^{1/2}$$

Back substituting:

$$\overline{C_{esc}^2} = \frac{\cancel{pA}}{\frac{\cancel{pA}}{4}\left(\frac{8k_BT}{\pi m}\right)^{1/2}} \cdot \left(\frac{2k_BT}{m}\right)^{3/2}\frac{1}{\sqrt{\pi}}$$

$$= 4\left(\frac{\cancel{\pi m}}{\cancel{8k_BT}}\right)^{1/2}\left(\frac{\cancel{2}k_BT}{m}\right)^{\cancel{3/2}\,2}\frac{1}{\cancel{\sqrt{\pi}}}$$

$$= 4\frac{k_BT}{m}$$

$$\therefore \boxed{\overline{C_{esc}^2} = \frac{4k_BT}{m}} > \frac{3k_BT}{m} = \langle E \rangle_{Reservoir}$$

Result seems counter intuitive, but actually makes sense. Molecules with a larger energy move at a higher speed, and scatter off the reservoir walls more frequently. This makes them more probable to hit the pinhole and escape, hence the larger average energy.

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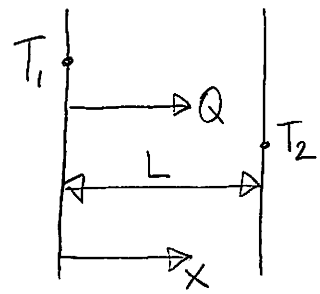
## Conduction

$\vec{V} = 0$, 1-D, Steady State, $\dot{Q}''' = 0$, $k$ = constant

We know that $\nabla^2 T = 0$ (heat equation)

① Slab




$Q$ = heat transfer rate [W]

Using our heat equation

$$\int \frac{\partial^2 T}{\partial x^2} = \int 0$$

$$\int \frac{\partial T}{\partial x} = \int C_1$$

$$T(x) = C_1 x + C_2 \Rightarrow B.C.'s \Rightarrow T(x=0) = T_1$$
$$T(x=L) = T_2$$

$$T(x=0) = T_1 = C_1(0) + C_2 \Rightarrow C_2 = T_1$$

$$T(x=L) = T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$$

$$\boxed{T(x) = \frac{T_2 - T_1}{L} x + T_1}$$

$\Rightarrow$ Temperature profile in the solid. Linear and decreasing from hot to cold

So if we define a heat transfer resistance; $R$, $A$ = area

Aside: Flow of Current: $I = \frac{\Delta V}{R}$

$$Q = \frac{\Delta T}{R} \Rightarrow Q = -kA\frac{\partial T}{\partial x} \Rightarrow \text{from above } \frac{\partial T}{\partial x} = \frac{T_2 - T_1}{L}$$

$$\frac{\partial T}{\partial x} = -\frac{(T_1 - T_2)}{L} = -\frac{\Delta T}{L} \Rightarrow \text{Back substituting}$$

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$$Q = \cancel{-}kA\left(\cancel{-}\frac{\Delta T}{L}\right)$$

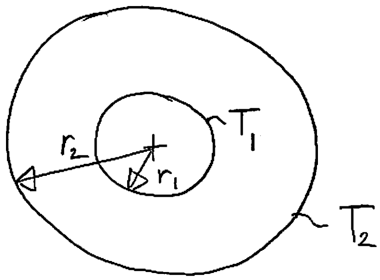
$$= \underbrace{\frac{kA}{L}}_{\frac{1}{R}} \cdot \Delta T$$

$$\frac{1}{R} \Rightarrow \boxed{R_{SLAB} = \frac{L}{kA}} \Rightarrow$$ Thermal Resistance of a solid slab.

$L$ = slab thickness
$A$ = heat transfer area
$k$ = thermal conductivity

*(Read the interesting paper about Fourier)

② Circular cylinder




We know our heat equation in radial coordinates is:

$$\cancel{\frac{1}{r}} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = 0$$

$$\int \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) = \int 0$$

$$r \frac{\partial T}{\partial r} = C_1$$

$$\int \frac{\partial T}{\partial r} = \int \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

Applying our boundary conditions:

$$T_1 = C_1 \ln r_1 + C_2 \quad ①$$

$$T_2 = C_1 \ln r_2 + C_2 \quad ②$$

Subtract ① & ② to get

$$C_1 = \frac{T_1 - T_2}{\ln r_1 - \ln r_2} = \frac{T_1 - T_2}{\ln (r_1/r_2)}$$

Back substitute into ①

$$C_2 = T_1 - \frac{(T_1 - T_2)}{\ln (r_1/r_2)} \ln (r_1)$$

Back substituting into our initial equation $T(r)$

$$T = \frac{T_1 - T_2}{\ln (r_1/r_2)} \ln r + T_1 - \frac{(T_1 - T_2) \ln r_1}{\ln (r_1/r_2)}$$

Finally:

$$\frac{T - T_1}{T_1 - T_2} = \frac{\ln (r/r_1)}{\ln (r_1/r_2)} \quad \text{or} \quad \boxed{\frac{T - T_1}{T_2 - T_1} = \frac{\ln (r/r_1)}{\ln (r_2/r_1)}}$$

↳ Radial temperature profile.

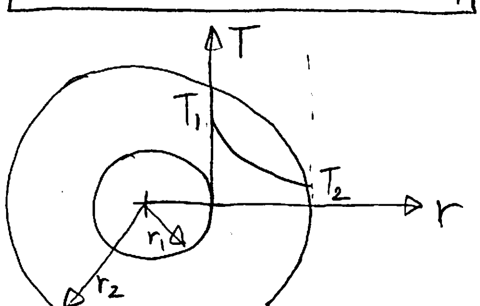
Calculating the thermal resistance:

$$Q = \frac{\Delta T}{R}$$

$$= -kA \left.\frac{\partial T}{\partial r}\right|_r = -k\,(2\pi \cancel{r} L)\, \frac{\overbrace{T_2 - T_1}^{\Delta T}}{\ln (r_2/r_1)} \cdot \frac{1}{\cancel{r}}$$

$$\boxed{R_{cyl} = \left| \frac{\ln (r_2/r_1)}{2\pi k L} \right|}$$

⇒ Cylindrical thermal resistance
Note the absolute value on the resistance. Doesn't matter if $r_2 > r_1$ or $r_1 > r_2$ ⇒ resistance is the same.




END OF LECTURE 3