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ME 521 - Convective Heat Transfer

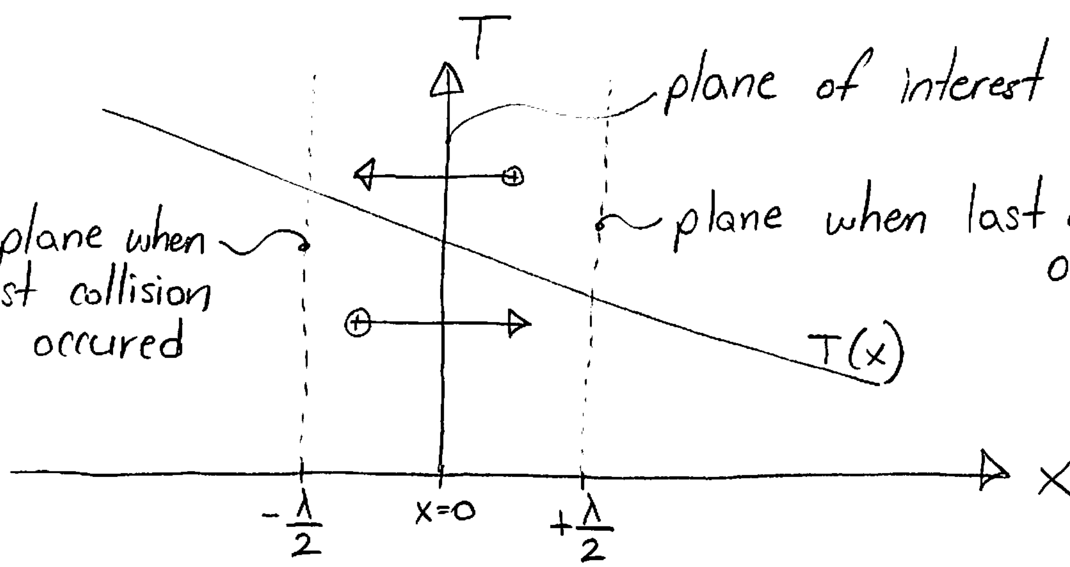
Mechanisms of heat transfer — Conduction = Convection w/ fluid motion
— Radiation

So to understand convection, we need to know a little bit about what conduction is.

Conduction is a process governed by molecular diffusion. Diffusion is the process by which molecules/atoms/electron intermingle as a result of their kinetic energy of random motion.

So wherever we have gradients, we have diffusional transport.

Let's take a look at heat:



Let's look at the energy crossing our plane at x=0.

$$q''_{x+} = \underbrace{n m \bar{c}}_{[\#/m^3]\cdot[kg/particle][m/s] = [kg/m^2 s]} \cdot C_p T\left(x - \frac{\lambda}{2}\right) \quad \left[\frac{J}{kg\cdot K}\right][K] = \left[\frac{J}{kg}\right]$$

$$\left[\frac{kg}{m^2 s}\right]\left[\frac{J}{kg}\right] = \frac{J}{m^2 s} = [W]$$

Just to be clear; $n = \#$ of molecules$/m^3$ $[m^{-3}]$
m = mass per molecule [kg]
C_p = specific heat capacity of molecule
$\bar{c}$ = average speed of molecule [m/s]

We can also say that $n \cdot m = \rho$
So now if we do the balance

$$\underbrace{\text{Energy Transfer (NET)}}_{q''_{NET,x}} = \underbrace{\text{Energy in } +x}_{q''_{x+}} - \underbrace{\text{Energy in } -x}_{q''_{x-}}$$

$$q''_{NET,x} = \rho C_p \bar{c}\left[T\left(x - \frac{\lambda}{2}\right) - T\left(x + \frac{\lambda}{2}\right)\right]$$
$$= -\rho C_p \bar{c}\left[T\left(x + \frac{\lambda}{2}\right) - T\left(x - \frac{\lambda}{2}\right)\right]$$

Multiplying by λ/λ

$$q''_{NET,x} = -\rho C_p \bar{c} \lambda \underbrace{\left[\frac{T\left(x + \frac{\lambda}{2}\right) - T\left(x - \frac{\lambda}{2}\right)}{\lambda}\right]}_{\frac{\partial T}{\partial x}}$$

$$\boxed{q''_{NET,x} = -\rho C_p \bar{c} \lambda \frac{\partial T}{\partial x}}$$

↳ Diffusional Transport is indeed gradient dependent.

We also see that: $\boxed{k = \rho C_p \bar{c} \lambda}$ ⇒ Note, in real life $\bar{c} = f(T)$

↳ Thermal conductivity

You can actually do this same derivation with momentum to derive viscosity (μ).

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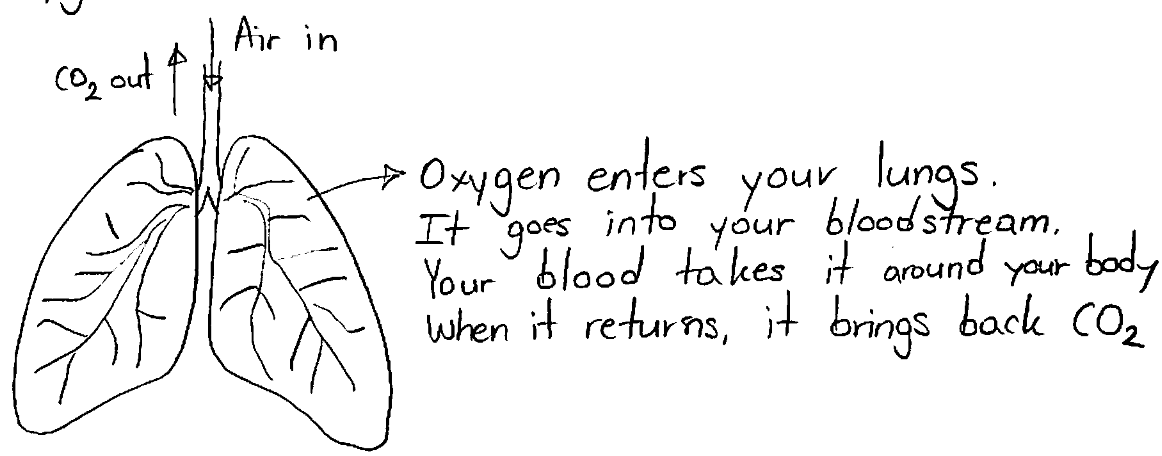
So what's the big deal? Well, I won't bore you with the obvious examples of enhanced heat transfer & mass transfer. It's fairly trivial that adding bulk fluid motion to help transport the mass, momentum, energy will enhance the process.

Let's talk a little more about nature & how we breathe.

Humans & animals have lungs to help them transport air (oxygen) from the outside to their bloodstream.



As living things, our cells produce energy, & need oxygen to do so. Viruses don't do this. That is what separates us from them.

Anyways, as we breate, we are using convection.
Note, convection = diffusion + advection
We force the fluid to come into our lungs & then interact with blood & exchange molecules.
Furthermore, our bloodstream is a very efficient convection mechanism. Blood flows through your body & to the cells that need oxygen.

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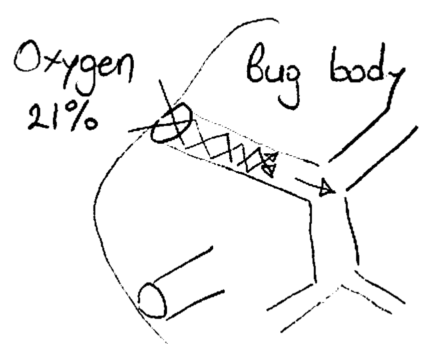
So you might say "So what, this is not a biology class". Well, have you ever thought about how bugs or insects breathe?

Bugs don't have lungs or a bloodstream, so they actually rely on molecular diffusion. They have small tiny openings all over their bodies. Oxygen from the air diffuses through the openings & goes right to the cell walls where it is used.



This mass transfer mechanism is limited by the amout of diffusional resistance through the channel. Hence, bugs are usually very small (diffusion lengths are small).

Note 300 million years ago, $C_{O_2} = 35\%$ as compared to $C_{O_2} = 21\%$ today. Hence prehistoric bugs were much much bigger!

Hence convection is very important! If you didn't have it (i.e. lungs), you would be much smaller.

For more info: noticing.co/how-insects-breathe/

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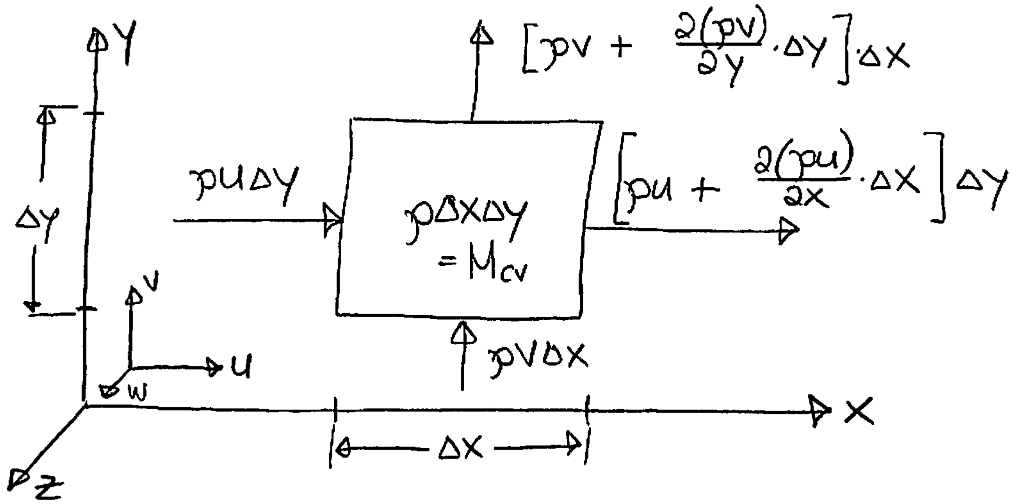
Fundamental Principles

Mass Conservation

We recall from engineering thermodynamics that:

$$\underbrace{\frac{\partial M_{cv}}{\partial t}}_{M_{cv}\text{ is instantaneous mass in control volume}} = \sum_{inlet} \dot{m} - \underbrace{\sum_{outlet} \dot{m}}_{\text{mass flow rates out of CV}}$$

Let's draw our control volume: (in 2D)



Writing out our mass ballance:

$$\frac{\partial}{\partial t}(\rho \Delta x \Delta y) = \rho u \Delta y + \rho v \Delta x - \left[\rho u + \frac{\partial(\rho u)}{\partial x} \cdot \Delta x\right] \Delta y - \left[\rho v + \frac{\partial(\rho v)}{\partial y} \cdot \Delta y\right] \Delta x$$

Divide through by $\Delta x \Delta y$ (constants) & let $\Delta x \to 0$, $\Delta y \to 0$

I added the third dimension arbitrarily.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

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Expanding with the product rule:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

or

$$\boxed{\frac{D\rho}{Dt} + \rho\nabla\cdot V = 0}$$ ⇒ V is velocity vector (u, v, w)

↳ Conservation of mass

and $$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

↳ Material derivative

Note for most convection problems encountered by engineers, the density variations in the flow are much smaller than local variations in velocity.

Thus ⇒ $\frac{D\rho}{Dt} \approx 0$ and: $$\boxed{\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0}$$

It is important to note that the simplified formulation applies to more than just incompressible fluids. For example, gases still follow it. The better distinction is the one outlined above $(\nabla\rho \ll \nabla V)$

For cylindrical and spherical:

$$\boxed{\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0}$$ ⇒ Cylindrical

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial \phi}\left(V_\phi \sin\phi\right) + \frac{1}{\sin\phi}\frac{\partial V_\theta}{\partial \theta} = 0}$$

↳ Spherical

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Momentum Conservation (Force Balance)

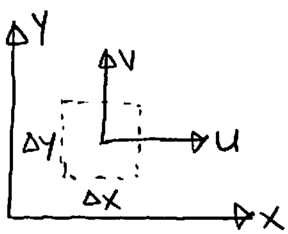
Recall from fluid mechanics (Newton's second law)

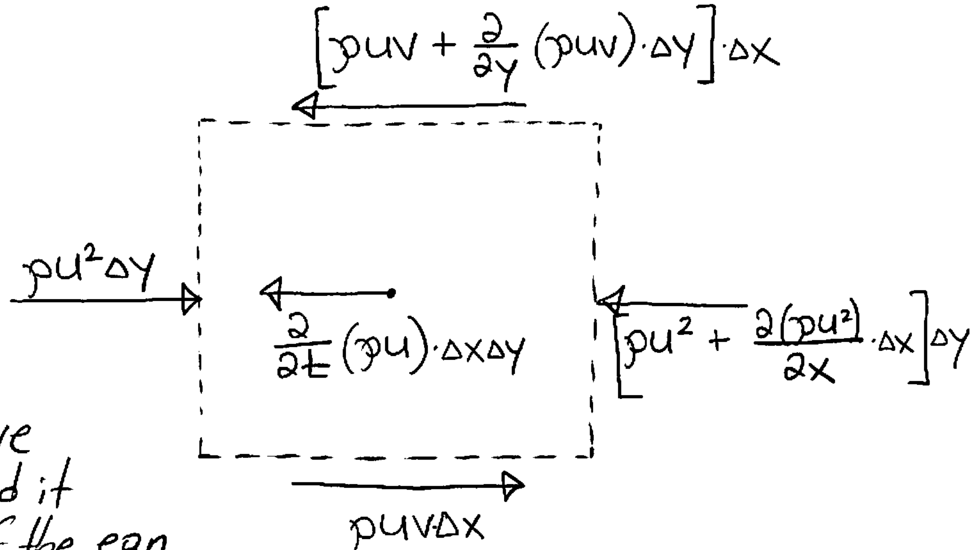
$$\underbrace{\frac{\partial}{\partial t}(M v_n)_{cv}}_{m\vec{a}\text{ term}} = \underbrace{\sum F_n}_{\text{Sum of forces}} + \underbrace{\sum_{inlet} \dot{m} v_n - \sum_{outlet} \dot{m} v_n}_{\text{Forces generated by inflow \& outflows.}}$$

where n denotes the direction of analysis, and v_n & F_n are the projected velocity & forces in the n direction.

Lets now analyze a control volume like before with a force balance in the x-direction only.

Looking first at the reaction forces due to momentum flow:





*Note, the transient term $(\frac{\partial}{\partial t})$ is negative because we've moved it to the other side of the eqn.

We can do the same for the forces acting on the CV, where we have normal stress (σ_x), tangential stress (τ_{xy}), and x-direction body force per unit volume (X).

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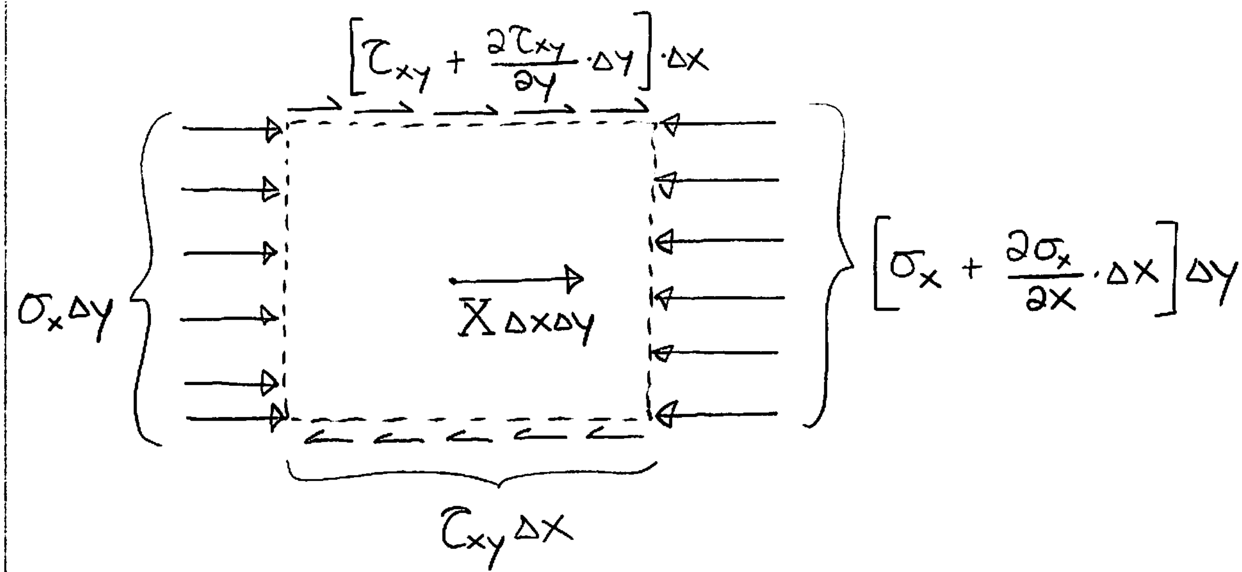

Writing out our complete momentum balance:

$$
\begin{aligned}
&-\frac{\partial}{\partial t}(\rho u \Delta x \Delta y) + \rho u^2 \Delta y - \left[\rho u^2 + \frac{\partial}{\partial x}(\rho u^2)\Delta x\right]\Delta y \\
&\quad + \rho u v \Delta x - \left[\rho u v + \frac{\partial}{\partial y}(\rho u v)\Delta y\right]\Delta x \\
&\quad + \sigma_x \Delta y - \left[\sigma_x + \frac{\partial \sigma_x}{\partial x} \cdot \Delta x\right]\Delta y - \tau_{xy}\Delta x \\
&\quad + \left[\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}\Delta y\right]\Delta x + X \Delta x \Delta y = 0
\end{aligned}
$$

Dividing through by $\Delta x \cdot \Delta y$, & letting $\Delta x, \Delta y \to 0$

$$
\rho \frac{Du}{Dt} + u\left[\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = -\frac{\partial \sigma_x}{\partial x} + \frac{\tau_{xy}}{\partial y} + X
$$

We know from mass conservation that the term in the brackets is zero.

$$
\rho \frac{Du}{Dt} = -\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X
$$

Now we relate the stresses σ_x and τ_{xy} to the local flow field by constitutive relations

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Viscosity, found by experiment.

$$\sigma_x = p - 2\mu \frac{\partial u}{\partial x} + \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

Scalar viscosity coefficient found by experiments.

These relations are emperical in origin. We define μ as the fluid viscosity. Note, these are valid for a Newtonian fluid which specifies the following conditions:

1) When at rest the stress is <u>hydrostatic</u> and the pressure inside the fluid is the thermodynamic press.
2) The stress tensor σ_{ij} is <u>linearly</u> proportional to D_{kl} (the deformation tensor).
3) No shear force may act during <u>solid body rotation.</u>
4) There are <u>no preferred directions</u> in the fluid, so fluid properties are point properties.

These approximate water, air, and many other fluids.

Substituting back our constitutive equations:

$$\boxed{\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[2\mu \frac{\partial u}{\partial x} - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + X}$$

↳ Navier-Stokes Equation (the real one!)

A usefull approximation for us is to assume incompressible and μ is constant. Then:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ (from mass conservation of an incompressible fluid)