

Turbulent Internal Flow Heat Transfer

Writing out our energy equation for fully developed flow,

$$\rho c_p \bar{u} \frac{\partial \bar{T}}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} (r q''_{app}) ; \quad q''_{app} = -k \frac{\partial \bar{T}}{\partial r}$$

Integrating from 0 to r and 0 to r_0

$$\int_0^r \rho c_p \bar{u} \frac{\partial \bar{T}}{\partial x} r dr = \int_0^r \partial (r q''_{app}) = r q''_{app} \quad (1)$$

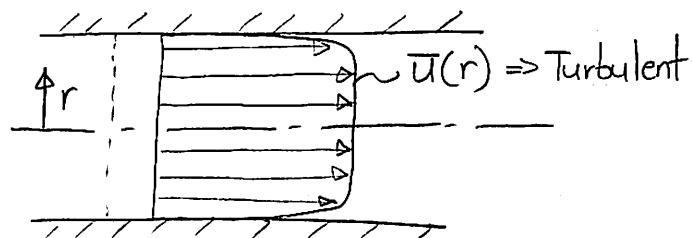
$$\int_0^{r_0} \rho c_p \bar{u} \frac{\partial \bar{T}}{\partial x} r dr = r_0 q''_0 \quad (2) \quad (q''_0 = \text{wall heat flux})$$

Dividing (1) & (2), we obtain

$$\boxed{\frac{q''_{app}}{q''_0} = M \left(1 - \frac{r}{r_0}\right)} ; \quad \boxed{M = \frac{\frac{2}{r^2} \int_0^r \bar{u} \frac{\partial \bar{T}}{\partial x} r dr}{\frac{2}{r_0^2} \int_0^{r_0} \bar{u} \frac{\partial \bar{T}}{\partial x} r dr}}$$

Note, if we have an x -independent heat-flux, $\frac{\partial \bar{T}}{\partial x} = f(r)$
Fully developed flow.

$$M = \frac{\frac{2}{r^2} \int_0^r \bar{u} r dr}{\frac{2}{r_0^2} \int_0^{r_0} \bar{u} r dr} \Rightarrow$$



Since $\bar{u}(r)$ is not a strong function of r (slug flow) in turbulent flows, then:

$$\frac{2}{r^2} \int_0^r \bar{u} r dr \cong \frac{2}{r_0^2} \int_0^{r_0} \bar{u} r dr \Rightarrow \boxed{M \cong 1}$$

Hence:

$$\boxed{\frac{q''_{app}}{q''_0} = \left(1 - \frac{r}{r_0}\right)} \quad \begin{matrix} \text{Same} \\ \text{as} \\ \Leftrightarrow \end{matrix} \quad \boxed{\frac{C_{app}}{C_0} = \left(1 - \frac{r}{r_0}\right)}$$

Remembering that: $\frac{\tau_{app}}{\tau_0} = \left(1 - \frac{y}{r_0}\right) \Rightarrow \frac{\tau_{app}}{\tau_0} = \frac{q_{app}''}{q_0''}$ (3)

$$\tau_{app} = (\mu + p\varepsilon) \frac{\partial \bar{u}}{\partial r} = p(\nu + \varepsilon) \frac{\partial \bar{u}}{\partial r} \quad (4)$$

$$-q_{app}'' = (k + p c_p \varepsilon_t) \frac{\partial \bar{T}}{\partial r} = p c_p (\alpha + \varepsilon_t) \frac{\partial \bar{T}}{\partial r} \quad (5)$$

$$\frac{p(\nu + \varepsilon) \frac{\partial \bar{u}}{\partial r}}{\tau_0} = \frac{-p c_p (\alpha + \varepsilon_t) \frac{\partial \bar{T}}{\partial r}}{q_0''} \Rightarrow \text{Sub (4) \& (5) into (3)}$$

$$\boxed{\frac{\nu + \varepsilon}{\tau_0} \frac{\partial \bar{u}}{\partial r} = \frac{c_p (\alpha + \varepsilon_t)}{q_0''} \frac{\partial \bar{T}}{\partial r}} \quad (6)$$

Now we can say that the pipe has 2 regions as drawn previously:
 $0 < y < y_1 \Rightarrow \nu \gg \varepsilon$ and $\alpha \gg \varepsilon_t$ ($y = r_0 - r$) (wall region)
 $y_1 < y < r_0 \Rightarrow \nu \ll \varepsilon$ and $\alpha \ll \varepsilon_t$ (core region - slug flow)

Integrate eq. (6) from 0 to y_1

$$\int_0^{y_1} \frac{\nu + \varepsilon}{\tau_0} d\bar{u} = \int_0^{y_1} \frac{c_p (\alpha + \varepsilon_t)}{q_0''} d\bar{T} \quad (\text{since wall region})$$

$$\frac{\nu}{\tau_0} \bar{u}_1 = \frac{c_p \alpha}{-q_0''} (\bar{T}_1 - T_0) \quad (7) \Rightarrow \bar{u}_1 \text{ and } \bar{T}_1 \text{ are time-averaged quantities at } y_1.$$

Now integrating from y_1 to y_2 where $\bar{u}(y_2) \approx U$ and $\bar{T}(y_2) \approx T_m$

$$\frac{\varepsilon}{\tau_0} (U - \bar{u}_1) = \frac{c_p \varepsilon_t}{-q_0''} (T_m - \bar{T}_1) \quad (8)$$

Eliminating \bar{u}_1 & \bar{T}_1 from (8) by subtracting (7) and knowing that:

$$f = \frac{\tau_0}{\frac{1}{2} \rho U^2} \text{ and } St = \frac{h}{(\rho c_p U)}, \text{ we obtain:}$$

$$\boxed{St = \frac{f/2}{Pr_t + (U_1/U)(Pr - Pr_t)}} \Rightarrow St = f(f, Pr_t, U_1/U) \quad (8D)$$

We can see again that $St \sim \frac{f}{2} \Rightarrow$ Colburn Analogy
 Note we don't know γ_1 & \bar{u}_1 or ϵ_t .
 Empirical results actually show:

$$\boxed{St Pr^{2/3} \cong \frac{f}{2}} \Rightarrow \text{Colburn Analogy still holds}$$

$$\Rightarrow Pr > 0.5; f = \frac{\tau_0}{\frac{1}{2} \rho U^2}$$

To be used in conjunction with the Moody chart to get f .

So for smooth pipes, we already showed that:

$$f = 0.046 Re_0^{-1/5} \quad (\text{Holds for } 2 \times 10^4 < Re_0 < 10^6)$$

$$St \cdot Pr^{2/3} = 0.023 Re_0^{-1/5} \Rightarrow St = \frac{Nu_0}{Re_0 \cdot Pr}$$

$$\frac{Nu_0 \cdot Pr^{2/3}}{Re_0 \cdot Pr} = 0.023 \cdot Re_0^{-1/5}$$

$$\boxed{Nu_0 = \frac{hD}{k} = 0.023 \cdot Re_0^{4/5} \cdot Pr^{1/3}} \Rightarrow \text{Turbulent flow}$$

$$\Rightarrow 2 \times 10^4 < Re_0 < 10^6$$

$$\Rightarrow Pr \geq 0.5$$

\hookrightarrow here $h = \text{constant}$, so need to use LMTD method: $Q = hA_w \Delta T_{LMTD}$

Some other famous ones include:

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\boxed{Nu_0 = 0.023 Re_0^{4/5} Pr^n} \Rightarrow \begin{cases} n = 0.4 & \text{for fluid being heated} \\ n = 0.3 & \text{for fluid being cooled} \end{cases}$$

\hookrightarrow Dittus-Boelter relation $0.7 < Pr < 120$
 $2500 < Re_0 < 1.24 \times 10^5$
 $L/D > 60$

Where temperature influence on properties is significant:

$$\boxed{Nu_0 = 0.027 Re_0^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_0}\right)^{0.14}} \Rightarrow 0.7 < Pr < 16,700$$

$$Re_0 > 10^4$$

$$\mu_0 \Rightarrow \text{Wall temp, } \mu \Rightarrow \text{Mean temp}$$

Most accurate:

$$\boxed{Nu_0 = \frac{(f/2)(Re_0 - 10^3)Pr}{1 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)}} \Rightarrow \text{For const. } q'' \text{ or } T_0$$

$$\Rightarrow \text{Gnielinski Correlation}$$

$$\Rightarrow \pm 10\% \text{ acc.} \quad (18)$$

Mass Transfer

Convection heat transfer typically accompanied by mass transfer.
 Atmospheric air \Rightarrow Driven by temperature differences \Rightarrow conveys water vapor

Ocean currents \Rightarrow Driven by $\Delta T \Rightarrow$ convey salt
 Chemical processing \Rightarrow reactive flows

Some basic definitions:

Imagine a container with a multi-component mixture (i.e. Air)

Volume, \forall mass, m multi-component

$m_i \equiv$ mass of individual component

$C_i = \frac{m_i}{\forall}$	\equiv concentration of component i in the mixture $[\text{kg}/\text{m}^3]$
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\Downarrow same thing

$\rho_i = \frac{m_i}{\forall}$	\equiv component density
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Since $\sum m_i = m$, then

$\rho = \sum C_i$	\equiv aggregate density (what we have been using so far)
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Note, the size of our fluid batch is described by extensive properties (m & \forall). Chemical engineers like to define a batch size by using the molar convention.

$1 \text{ Mole} \equiv 6.022 \times 10^{23} \text{ elementary entities}$	\equiv Avogadro's const.
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$M \equiv$ molar mass	\equiv mass of 1 Mole of a mixture or component
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$[\text{kg}/\text{mol}]$

So we can now say:

$$\boxed{n = \frac{m}{M}} \quad \text{and} \quad \boxed{n_i = \frac{m_i}{M_i}} \Rightarrow \text{Component}$$

↳ Mixture

In dimensionless form:

$$\boxed{\phi_i = \frac{m_i}{m}} \Rightarrow \text{Mass fraction} \quad (\sum \phi = 1)$$

or

$$\boxed{x_i = \frac{n_i}{n}}, \quad \boxed{\sum x_i = 1} \equiv \text{Mole Fraction}$$

So we have 2 dimensionless and 1 dimensional way to describe mass transfer. To relate these:

$$\boxed{C_i = \rho \phi_i = \rho \frac{M_i}{M} x_i} \quad \text{where} \quad \boxed{M = \sum M_i x_i}$$

Ideal Gases (high temperature, low pressure)

$$\boxed{PV = mRT} \quad \text{or} \quad \boxed{PV = n\bar{R}T} \quad \text{where} \quad \boxed{\bar{R} = 8.314 \text{ J/mol}\cdot\text{K}}$$

Here we can define partial pressure as:

$$\boxed{R = \frac{\bar{R}}{M}} \quad \downarrow \text{Universal Gas Constant}$$

$$\boxed{P_i V = m_i R T} \quad \text{or} \quad \boxed{P_i V = n_i \bar{R} T} = \text{pressure one would measure if } i \text{ were to fill volume } V \text{ at temperature } T.$$

Summing over i :

$$\boxed{P = \sum P_i} \equiv \text{Dalton's Law} \quad \text{or} \quad \boxed{x_i = \frac{P_i}{P}}$$

Note, this is all valid for equilibrium, homogeneous, mixtures.