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Colburn Analogy

Since we have an analogy between heat & mass transfer, we can use the Colburn analogy as well for external & internal flows.

$$St_x = \frac{1}{2} C_{f,x} Pr^{-2/3} \quad \text{(Heat transfer Colburn)}$$

Now we need the mass transfer analog to St_x:

$$St_x = \frac{h_x}{\rho C_p U_\infty} = \frac{h_x \alpha}{k U_\infty} = \frac{q_0'' \alpha}{(T_0 - T_\infty) k U_\infty}$$

For mass transfer: $q_0'' \rightarrow j_0$

$T_0 - T_\infty \rightarrow C_0 - C_\infty$

$\alpha \rightarrow D$

$k \rightarrow D$

$$\frac{q''\alpha}{(T_0 - T_\infty) k U_\infty} \longrightarrow \frac{j_0 \cancel{D}}{(C_0 - C_\infty)\cancel{D} U_\infty} = \frac{h_m \cancel{(C_0 - C_\infty)}}{\cancel{(C_0 - C_\infty)} U_\infty} = \frac{h_m}{U_\infty}$$

$\therefore$ $\boxed{St_m = \frac{h_m}{U_\infty}}$ $\equiv$ Local mass transfer Stanton #.

Now we can say our Colburn analogy becomes:

$$\boxed{St_m \cdot Sc^{2/3} = \frac{C_{f,x}}{2}} \Rightarrow \text{Mass Transfer Colburn analogy}$$

Taking this one step further, & relating our 2 analogies:

$$St_x Pr^{2/3} = St_m \cdot Sc^{2/3}$$

$$\frac{h}{\rho C_p \cancel{U_\infty}} \cdot Pr^{2/3} = \frac{h_m}{\cancel{U_\infty}} Sc^{2/3}$$

$\alpha \equiv$ mixture thermal diff.

$$\boxed{\frac{h}{h_m} = (\rho C_p)\left(\frac{\alpha}{D}\right)^{2/3} = \rho C_p Le^{2/3}} \Rightarrow \boxed{Le = \frac{Sc}{Pr} = \frac{\alpha}{D} \equiv \text{Lewis \#}}$$

So by measuring h_m, we can back calculate h!

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Turbulent Flow Mass Transfer

Using our same analogy, our solution becomes trivial

$$\overline{Nu_L} = 0.037 Pr^{1/3} Re_L^{4/5} \quad (Pr > 0.5,\ 10^6 < Re_L < 10^8)$$

$$\Downarrow$$

$$\boxed{\overline{Sh_L} = \frac{\overline{h}_m L}{D} = 0.037 Sc^{1/3} Re_L^{4/5}} \quad (Sc > 0.5,\ 10^6 < Re_L < 10^8)$$

↳ Flat Plate, turbulent, incompressible

For a cylinder in turbulent cross flow:

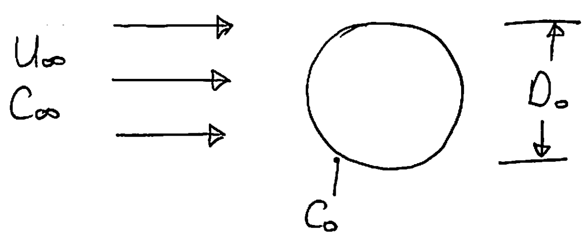
$$\boxed{\overline{Sh_{D_o}} = \frac{\overline{h}_m D_o}{D} = 0.3 + \frac{0.62 Re_{D_o}^{1/2} Sc^{1/3}}{\left[1+(0.4/Sc)^{2/3}\right]^{1/4}} \cdot \left[1+\left(\frac{Re_{D_o}}{282,000}\right)^{5/8}\right]^{4/5}}$$

$Re_{D_o} Sc > 0.2$

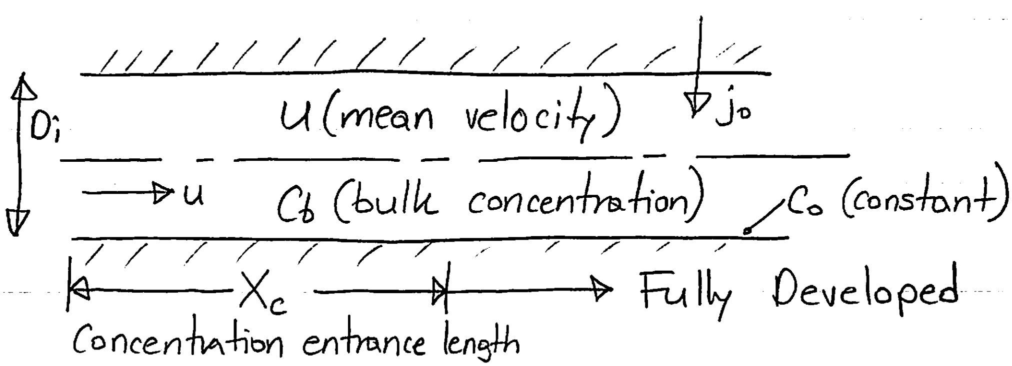
For a sphere in turbulent cross flow:

$$\boxed{\overline{Sh_{D_o}} = 2 + \left(0.4 Re_{D_o}^{1/2} + 0.06 Re_{D_o}^{2/3}\right) Sc^{0.4}} \quad (3.5 < Re_{D_o} < 7.6\times10^4)$$

Note, our cylinder and sphere cases look like:



Internal Forced Convection Mass Transfer



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Note, our analogies still hold for internal flow:

$$\boxed{\frac{X_c}{D_h} \cong 0.04\, Re_{D_h} \cdot Sc} \Rightarrow \text{Laminar Flow } (Re_{D_h} < 2300)$$

$$Re_{D_h} = \frac{u D_h}{\nu}$$

$$\boxed{C_b = \frac{1}{UA}\int_A u C\, dA} \equiv \text{Bulk concentration of the stream}$$

$$\boxed{Sh_{D_i} = \frac{h_m D_i}{D} = 3.66} \Rightarrow \begin{array}{l} D_i = \text{tube inner diameter (pipe)} \\ D = \text{mass diffusivity} \end{array}$$

$\longrightarrow$ Laminar pipe flow. ($Re_{D_h} < 2300$)

For <u>turbulent pipe flow</u>:

$$\boxed{\frac{X_c}{D_h} \approx 10} \Rightarrow \text{Concentration entrance length in turbulent flow.}$$

Using our mass transfer analogy;

$$\boxed{Sh_{D_i} = 0.023\, Re_{D_i}^{4/5}\, Sc^{1/3}} \quad (Sc > 0.5,\ 2\times10^4 < Re_{D_i} < 10^6)$$

In general, we should always remember:

Heat Transfer	Mass Transfer
$h\ [W/m^2 \cdot K]$	$h_m \left[\frac{m}{s}\right] = \left[\frac{kg}{m^2 \cdot s \cdot kg/m^3}\right]$
Re (Reynolds)	Re (Reynolds)
$Nu = \frac{hL}{k}$ (Nusselt)	$Sh = \frac{h_m L}{D}$ (Sherwood)
$Nu = f(Re, Pr)$	$Sh = f(Re, Sc)$
$Ra = \frac{g\beta\Delta T L^3}{\nu\alpha}$ (Rayleigh)	$Ra_m = \frac{g\Delta\rho L^3}{\mu D}$ (Rayleigh mass transf #)

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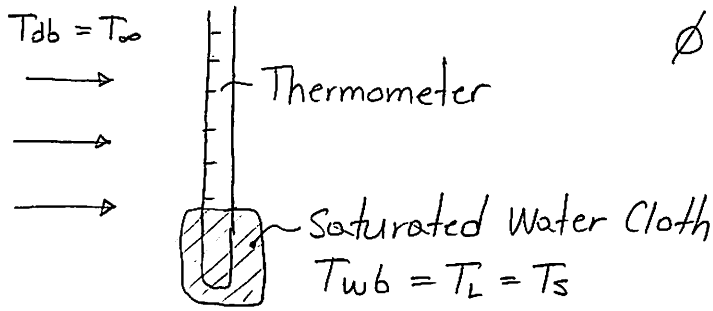
Wet Bulb Psychrometer (Note, $T_{wb} \neq T_{dew\ point}$)!

A simple device used to measure the relative humidity of air.



$\phi \equiv$ Relative Humidity

$$\boxed{\phi = \frac{P_{H_2O,\infty}}{P_{H_2O,SAT}(T_\infty)}}$$

↳ Need a way to determine $P_{H_2O,\infty}$

Note, we defined on pg. 182 that $C_i = \rho_i \equiv$ component density

So if we do an energy balance on our psychrometer water cloth in equilibrium (evap. = convective heating)

$$Q_{EVAP} = hA(\overset{T_\infty}{T_{db}} - T_{wb}) \quad (1)$$

But we also know that evaporation $\sim h_{fg}$:

$$Q_{EVAP} = \dot{m} h_{fg} = h_m A (C_{H_2O,s} - C_{H_2O,\infty}) \cdot h_{fg} \quad (2)$$

Where $h_{fg} \equiv$ latent heat of phase change

Since $C_{H_2O,s} = \rho_{H_2O,s}$ and $C_{H_2O,\infty} = \rho_{H_2O,\infty}$
And using our ideal gas law for air/water vapor mixture

$$\rho_{H_2O,\infty} = \frac{P_{H_2O,\infty}}{R_{H_2O} \cdot T_\infty} \quad (3)$$

$$\rho_{H_2O,s} = \rho_{H_2O,SAT}(T_{wb}) = \frac{P_{H_2O,s}}{R_{H_2O} \cdot T_{wb}} \quad (4)$$

$\Rightarrow$ Note the boundary condition.

Rearanging & back substituting (1)-(4):

$$\underbrace{\left(\frac{h}{h_m}\right)}_{R_{H_2O} \cdot \rho C_p Le^{2/3}} \frac{(T_\infty - T_{wb})}{h_{fg}} = \rho_{H_2O,s} - \rho_{H_2O,\infty}$$

We know everything except $P_{H_2O,\infty}$.

$$R_{H_2O} \cdot \rho C_p Le^{2/3} \cdot \frac{(T_\infty - T_{wb})}{h_{fg}} = \frac{P_{H_2O,s}}{T_{wb}} - \frac{P_{H_2O,\infty}}{T_\infty}$$

$\Rightarrow$ Can solve explicitly for $P_{H_2O,\infty}$ & ϕ.

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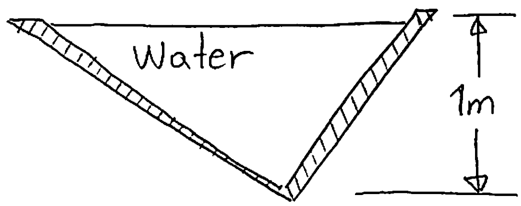
Ex #1| A channel 25m long & 1m deep used for storage of water. Water & surroundings are at 25°C and $RH = \phi = 50\%$.



a) If air moves at 5m/s along the channel length, determine the rate of water loss due to evaporation.

First we need to determine the flow regieme:

$$Re_L = \frac{U_\infty L}{\nu_{air}} = \frac{(5\,m/s)(25\,m)}{(15.66\times10^{-6}\,m^2/s)} = 7.98\times10^6 > 5\times10^5$$

Turbulent Flow

We can now use our analogies to solve. (Turbulent correlations)

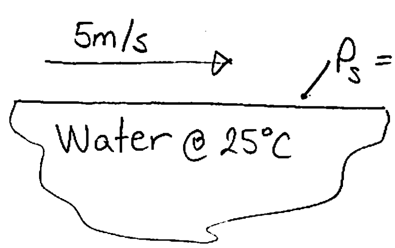
$$\overline{Nu_L} = 0.037\,Pr^{1/3}\,Re_L^{4/5} \quad (Pr \geq 0.5,\ 10^6 < Re_L < 10^8)$$

$$\bar{h} = \frac{k}{L}\cdot 0.037\,Pr^{1/3}\,Re_L^{4/5} = \frac{(0.0267\,W/m\cdot K)}{25\,m}\cdot 0.037(0.69)^{1/3}\cdot(7.98\times10^6)^{4/5}$$

$$\bar{h} = 11.6\ W/m^2\cdot K$$

We know that $\dot{m} = \bar{h}_m A(\rho_s - \rho_\infty)$

Looking up our water vapor densities:



$\rho_s = \rho_{sat}(25°C) \Rightarrow \rho_s = 0.02282\,kg/m^3$

$\rho_\infty = 0.5\,\rho_{sat}(25°C) = 0.01141\,kg/m^3$

Now we can solve for $\bar{h}_m$

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We showed before that:

Remember, ρ & C_p are our mixture properties (air)

$$\frac{\bar{h}}{\bar{h}_m} = \rho C_p Le^{2/3} \quad ; \quad Le = \frac{Sc}{Pr} \quad ; \quad h_{fg} = 2.442\times10^{6}\, J/kg$$

Remember, ν is the mixture property, so ν_{air}.

$$Sc = \frac{\nu}{D} \Rightarrow D_{H_2O-Air} = 22\times10^{-6}\, m^2/s \quad \text{(From Table 11.1, pg. 187)}$$

$$Sc_{Air} = \frac{15.66\times10^{-6} m^2/s}{22\times10^{-6} m^2/s} = 0.7118$$

$$Le = \frac{Sc}{Pr} = \frac{0.7118}{0.69} = 1.0316$$

Back substituting into our analogy

$$\bar{h}_m = \frac{\bar{h}}{\rho C_p Le^{2/3}} = \frac{11.6\, W/m^2\cdot K}{(1.177 kg/m^3)(1005 J/kg\cdot K)(1.036)^{2/3}}$$

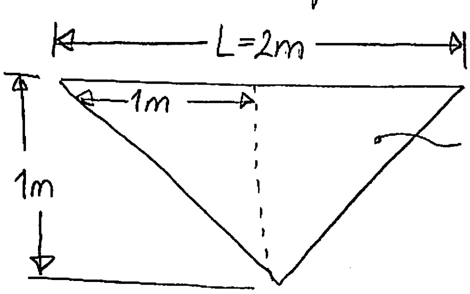
$$\bar{h}_m = 0.007675\ m/s$$

Now we can calculate $\dot{m}$ and our mass loss:

$$\dot{m} = \bar{h}_m A(\rho_s - \rho_\infty) = (0.007675\, m/s)(50 m^2)(0.02282 - 0.01141)$$

$$\boxed{\dot{m} = 0.00438\, kg/s = 15.76\, kg/hour}$$

b) Obtain an expression for the rate of water depth decrease due to evaporation. How long would it take to empty the tank due to evaporation?



$$\forall = \frac{bhL}{2} \quad , \quad h = b \text{ (for 1 half)}$$

$$\forall = \frac{h^2 L}{2}$$

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ρ_{water} since pure water in the tank.

$$\rho_w \frac{\partial \forall}{\partial t} = \dot{m}$$

$$\frac{\dot{m}}{\rho_w} = \frac{\partial \forall}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{h^2 L}{2}\right) = \frac{1}{2}\cdot\frac{L}{2}\frac{\partial}{\partial t}h^2 = \frac{1}{\cancel{2}}\cdot\frac{L}{2}\cdot\cancel{2}h\cdot\frac{\partial h}{\partial t}$$

$$\int_0^H h\,dh = \int_0^t \frac{2\dot{m}}{\rho_w L}\,dt$$

$$\frac{H^2}{2} = \frac{2\dot{m}}{\rho_w L}t$$

$$\boxed{t = \frac{H^2 \rho L}{2\dot{m}}} \Rightarrow t = \frac{(1m)^2(1000.0 kg/m^3)(25m)}{(2)(0.0052 kg/s)}$$

$$\boxed{t = 27.82 \text{ days}}$$

Note, this is incorrect since we can't assume $\dot{m}$ = constant. The right way to do it is:

$$\dot{m} = \bar{h}_m (2h) L (\rho_s - \rho_\infty)$$

$$\rho \frac{\partial \forall}{\partial t} = \rho \frac{\partial}{\partial t}\frac{h^2}{2}\cdot L = \bar{h}_m (2hL)(\rho_s - \rho_\infty)$$

$$\rho(\cancel{2h})\frac{1}{2}\frac{\partial h}{\partial t}\cdot\cancel{L} = \bar{h}_m(\cancel{2hL})(\rho_s - \rho_\infty)$$

$$\frac{dh}{dt} = \frac{2\bar{h}_m(\rho_s - \rho_\infty)}{\rho}$$

$$h(t) = \frac{2\bar{h}_m(\rho_s - \rho_\infty)}{\rho}\cdot t + C$$

$$h(0) = H \Rightarrow C$$

$$h(t) = H + \frac{2\bar{h}_m(\rho_s - \rho_\infty)t}{\rho} \Rightarrow \text{Let } h = 0 \text{ (when dry)}$$

$$0 = H + \frac{2\bar{h}_m(\rho_s - \rho_\infty)}{\rho}t$$

$$\boxed{t = 101.23 \text{ days}}$$